2019 Study trip in Ottawa, Mathematics in English

Kazuo AKIYAMA Eriko OHNO

<Summary>

This was the 5th Canada study trip. This time it was up to 33 students. Most of the schedule was the same as in the previous. There are two differences in the previous itinerary. One of them stopped their one-day trip to Montreal and instead turned to free time in Ottawa's. The other one is that the lecturer is back by Dr. Mingarelli. So, the problem this time was in the analysis field. Dr. Mingarelli was very busy this year and couldn't take enough time to create 10 problems. The problem received was not good enough for the students. I allowed to use math problems which were given by Dr. Mingarelli two years ago. Even if the problem was not very appropriate, I would like to express my heartfelt gratitude to Professor Mingarelli. Because I understand that it takes a lot of time and ability to create a new problems.

It is in the student's own mind whether 8 months were long or short after the preparation period. I am convinced that the act of thinking about mathematics in English, thinking about answers in English, and explaining them in English has immeasurable effects on students.

<10 problems, solution, presentation and students' impressions>

- 1. Define a straight-edge and compass construction in plane geometry. Give Archimedes' construction for the trisection of an angle and show why it is not an acceptable solution to this 2500years old problem. What type of triangle can be trisected (previous)
- 2. Collapsing Gas Clouds and Stability (previous)
- State and prove the classical Theorem of Pythagoras. Then state such a theorem for a SPHERICAL triangle. Can you prove it? Also, can you prove an equivalent theorem in HYPERBOLIC geometry? (previous)

4. a) Show that the *escape velocity*, v, of a small spherical object of mass 1 from the gravitational field of a body of a spherical mass M is given by

$$v = \sqrt{\frac{\mathrm{GM}}{\mathrm{R}}},$$

where R is the distance between the centers of the two bodies.

- b) Given the mass M, how small should the distance R be in order that the *escape velocity* v > c, where c is the speed of light?
- c) Conclude from b) that if the body M were a star then such a body would have to appear "black" when viewed from the outside. This is an example of a "black hole".
- d) How small should the earth be in order for it to appear "black" to an outside observer? (previous)
- 5. The Fibonacci sequence F_n is a sequence of numbers defined by the recurrence relation, F_{n+1} = F_n + F_{n-1} where F₀ = 0, F₁ = 1 and the first few terms are given by 0, 1, 1, 2, 3, 5, 8, 13, 21, ... (previous)
 - a) Show that if the following limit exists, then

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\varphi\,,$$

where $\varphi = 1:618...$ is the so-called *Golden Number*.

b) If α is any positive integer, show that Show that if the following limit exists, then

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\varphi^{\alpha}.$$

- 6. The Close Encounter to the Sun of Barnard's Star (previous)
- 7. Why are hot things red? (previous)
- 8. Show that if $\lim_{x \to a} x^{x^{x'}} = L$ where the number of exponents tends to ∞ , then

$$a < e^{\frac{1}{e}}$$

- 9. Show that $\forall n \ge 1$ $\lim_{|x| \to \infty} D^n(e^{-x^2}) = 0$
- 10. Two concentric ellipses are given by $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (E1) and $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (E2). Find the points on E2 obtained by the intersection of the tangent line to the point $\left(1, \frac{3\sqrt{3}}{2}\right)$ on E1.
- 11. Find the largest value of xy that x + y = 4 without using Calculus.
- 12. Two wides of radius 8cm and 2cm are tangent to each other. Find the length of their common external tangent.

13. What is wrong with the following proof.

We know that

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$
 for all $n \ge 1$

Replacing n by n-1, we get

$$1+2+3+\dots+(n-1)=\frac{(n-1)n}{2}\dots *$$

Adding 1 to both sides of * gives

$$1+2+3+\dots+n = \frac{(n-1)n}{2}+1$$

Therefore,

$$\frac{n(n+1)}{2} = \frac{(n-1)n}{2} + 1$$
$$\rightarrow n = 1$$

- 14. An isosceles triangle has base equal to 6cm and height equal to 4cm. Find the radius"*r*" of the inscribed circle.
- 15. The proof of Leonardo's theorem

		男-20、女-18			班	問題番号	
D	40	要藤 啓一郎	Keiichiro YODO	男	1	8	
Е	34	濱田 南帆	Minaho HAMADA	女	1	9	
G	З	石本 貴昭	Takaaki ISHIMOTO	男	1	8	
Ι	27	*藤澤 麻央	Mao FUJISAWA	女	1	9	
I	40	吉池 百栞	Moka YOSHIIKE	女	1	9	
В	20	関口 瑠輝	Ryuki SEKIGUCHI	男	2		
D	10	*佐久間 駿	Shun SAKUMA	男	2	1	
Н	27	徳光 さゆり	Sayuri TOKUMITSU	女	2		
Е	16	*坂 朝日	Asahi SAKA	男	3		
I.	8	落畑 涼太	Ryota OCHIHATA	男	3	7	
Н	29	中島 杏	Ann NAKAJIMA	女	3		
А	20	鈴木 希彩	Maaya SUZUKI	女	4	10	
С	21	髙橋 侑希	Yuki TAKAHASHI	男	4	12	
G	40	柳下 実輝	Miki YAGISHITA	女	4	10	
Ι	15	*小池 創大	Sodai KOIKE	男	4	11	
В	17	小峯 栞緒	Shion KOMINE	女	5		
С	22	滝川 聖吾	Seigo TAKIKAWA	男	5	c	
Е	13	小牧 真斗	Manato KOMAKI	男	5	6	
Е	29	*二階堂 里菜	Rina NIKAIDO	女	5		
В	21	瀬越 貴久	Takahisa SEGOE	男	6		
С	37	三川 和真	Kazuma MITSUKAWA	男	6		
G	16	*櫻田 翔子	Shoko SAKURADA	女	6	2	
I	26	野田 みなつ	Minatsu NODA	女	6		
С	30	野田奈菜子	Nanako NODA	女	7		
D	28	*原田 誠也	Seiya HARADA	男	7		
Е	36	福田 紘夢	Hiromu FUKUDA	男	7	3	
Ι	З	上原 千晴	Chiharu UEHARA	女	7		
Е	28	中村 万智	Machi NAKAMURA	女	8	13	
F	21	*瀬野 来海	Kurumi SENO	女	8	13	
Н	25	髙宮 丈	Jo TAKAMIYA	男	8	15	
Н	41	山田 壮太	Sota YAMADA	男	8	14	
В	18	佐藤 萌愛	Moe SATO	女	9		
Е	42	八束 彬皓	Akihiro YATSUDUKA	男	9		
G	З	石本 貴昭	Takaaki ISHIMOTO	男	9	4	
Ι	37	*山岡 千紘	Chihiro YAMAOKA	女	9		
D	1	五十嵐 沙有実	Sayumi IGARASHI	女	10		
D	41	横井 駿東	Hayato YOKOI	男	10	5	
Ι	33	*丸山 貴人	Takato MARUYAMA	男	10		

The limit value of infinite exponent tower

Takaaki ISHIMOTO, Keiichiro YODO

Group 1

Question

Show that if $\lim_{x \to a} x^{x^{x'}} = L$ where the number of exponents tends to ∞ , then

$$a \leq e^{\frac{1}{e}}$$

Solution

Set the base of the natural logarithm e

Assume that $\lim_{x \to a} x^{x^{x'}}$ has the limit value, i.e. $\lim_{x \to a} x^{x^{x'}} = L$ Set

$$\lim_{x \to a} x^{x^{x^{*}}} = a^{a^{a^{*}}} \quad \text{where } a > 0$$

And set

$$t = a^{a^{a^{\cdot}}} = a^t \iff a = t^{\frac{1}{t}} \quad \text{where } t > 0$$

Take the natural logarithm of the both sides

$$\log a = \frac{\log t}{t}$$

Differentiate the both sides by t

$$\frac{d}{dt}(left \, side) = \frac{d}{dt} \log a$$
$$= \frac{d}{da} \log a \cdot \frac{da}{dt}$$
$$= \frac{1}{a} \cdot \frac{da}{dt}$$

Also

$$\begin{aligned} &\frac{d}{dt}(right \, side) \\ &= \frac{d}{dt} \left(\frac{\log t}{t} \right) \\ &= \frac{1}{t^2} \left(t \cdot \frac{d}{dt} \log t - \log t \cdot \frac{dt}{dt} \right) \left(\because \left(\frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2} \right) \\ &= \frac{1}{t^2} \left(t \cdot \frac{1}{t} - \log t \cdot 1 \right) = \frac{1 - \log t}{t^2} \end{aligned}$$

Therefore

$$\frac{d}{dt}(left \, side) = \frac{d}{dt}(right \, side)$$
$$\frac{1}{a} \cdot \frac{da}{dt} = \frac{1 - \log t}{t^2}$$
$$\Leftrightarrow \frac{da}{dt} = a \cdot \frac{1 - \log t}{t^2}$$
$$= t^{\frac{1}{t}} \cdot \frac{1 - \log t}{t^2} \left(\because a = t^{\frac{1}{t}} \right)$$
$$= t^{\frac{1}{t} - 2}(1 - \log t)$$

If $\frac{da}{dt} > 0$

$$1 - \log t > 0 \Leftrightarrow -\infty < \log t < 1$$
$$\Leftrightarrow 0 < t < e$$

t	0	•••	е	•••		
$\frac{da}{dt}$		+	0	_		
а		7	$e^{\frac{1}{e}}$	7		
$\therefore a = t^{\frac{1}{t}} \le e^{\frac{1}{e}} (Q. E. D.)$						

Table 1. The first derivative test of $a = t^{\frac{1}{t}}$

The following graph shows $a = t^{\frac{1}{t}}$.



The limit value of th derivative of e^{-x^2}

Mao FUJISAWA, Minaho HAMADA, Moka YOSHIIKE

Group 1

Question

Show that
$$\forall n \ge 1$$
 $\lim_{|x| \to \infty} D^n (e^{-x^2}) = 0$

Solution

Lemma 1

The coefficient of e^{-x^2} of $D^n(e^{-x^2})$ is the *n* th degree polynomial in *x*.

Proof

Lemma1 means below.

$$D^{n}(e^{-x^{2}}) = (a_{0} + a_{1}x + a_{2}x^{2} + \dots + a_{n-1}x^{n-1} + a_{n}x^{n})e^{-x^{2}}$$

where $\forall n \in \mathbb{N}, \exists a_{n} \in \mathbb{R}, a_{n} \neq 0$

Prove Lemma 1 by using Mathematical Induction.

(1) If n = 1

$$D(e^{-x^2}) = -2xe^{-x^2}$$

Thus, Lemma 1 is correct in the case n = 1.

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(2) If n = k

Assume that

$$D^{k}(e^{-x^{2}}) = (a_{0} + a_{1}x + a_{2}x^{2} + \dots + a_{k-1}x^{k-1} + a_{k}x^{k})e^{-x^{2}}$$

where $a_{k} \neq 0$

Then, prove Lemma 1 is correct for n = k + 1 if Lemma 1 is true for n = k, i.e.

$$= \{a_{1} + (a_{2} - 2a_{0})x + (a_{3} - 2a_{1})x^{2} + \cdots$$

$$D^{k+1}(e^{-x^{2}}) = D\{D^{k}(e^{-x^{2}})\}$$

$$= D\{(a_{0} + a_{1}x + a_{2}x^{2} + \cdots + a_{k-1}x^{k-1} + a_{k}x^{k})e^{-x^{2}}\}$$

$$= \{D(a_{0} + a_{1}x + a_{2}x^{2} + \cdots + a_{k-1}x^{k-1} + a_{k}x^{k})\}e^{-x^{2}}$$

$$+ (a_{0} + a_{1}x + a_{2}x^{2} + \cdots + a_{k-1}x^{k-1} + a_{k}x^{k})\{D(e^{-x^{2}})\}$$

$$= (a_{1} + a_{2}x + a_{3}x^{2} \cdots + a_{k-1}x^{k-2} + a_{k}x^{k-1})e^{-x^{2}}$$

$$- 2x(a_{0} + a_{1}x + a_{2}x^{2} + \cdots + a_{k-1}x^{k-1} + a_{k}x^{k})e^{-x^{2}}$$

$$= \{(a_{1} + a_{2}x + a_{3}x^{2} \cdots + a_{k-1}x^{k-2} + a_{k}x^{k-1})$$

$$- (2a_{0}x + 2a_{1}x^{2} + 2a_{2}x^{3} + \cdots + 2a_{k-2}x^{k-1} + 2a_{k-1}x^{k} + 2a_{k}x^{k+1})\}e^{-x^{2}}$$

$$\cdots + (a_{k} - 2a_{k-2})x^{k-1} - 2a_{k-1}x^{k} - 2a_{k}x^{k+1}\}e^{-x^{2}}$$

By Mathematical Induction, Lemma 1 is correct for all $n \in \mathbb{N}$

Lemma 2

$$\lim_{x \to \infty} \frac{x}{e^x} = 0$$

Proof

Assume that x is large enough.

$$0 < 2 < e$$

$$\Leftrightarrow 0 < 2^{x} < e^{x}$$

$$\Leftrightarrow 0 < \frac{1}{e^{x}} < \frac{1}{2^{x}}$$

$$\Leftrightarrow 0 < \frac{x}{e^{x}} < \frac{x}{2^{x}}$$

Also

$$0 < |x| \le x$$

$$\Leftrightarrow 0 < 2^{|x|} \le 2^{x}$$

$$\Leftrightarrow 0 < \frac{1}{2^{x}} \le \frac{1}{2^{|x|}}$$

$$\Leftrightarrow 0 < \frac{x}{2^{x}} < \frac{|x|+1}{2^{|x|}} \quad (\because x < |x|+1)$$

Here, |x| is the floor function (the function which output the greatest integer less than or equal to a real number x).

Then

$$0 < \frac{x}{e^x} < \frac{x}{2^x} < \frac{|x|+1}{2^{|x|}}$$

By using the Binomial Theorem

$$2^{|x|} = (1+1)^{|x|}$$

$$= \sum_{k=0}^{|x|} C_k 1^k 1^{n-k}$$

$$= \sum_{k=0}^{|x|} C_k$$

$$= |x| C_0 + |x| C_1 + |x| C_2 + \dots + |x| C_{|x|} > |x| C_2 = \frac{x \cdot (x \cdot -1)}{2} > 0$$

$$\Leftrightarrow 0 < \frac{1}{2^{|x|}} < \frac{2}{|x|(|x|-1)}$$

$$\Leftrightarrow 0 < \frac{|x|+1}{2^{|x|}} < \frac{2(|x|+1)}{|x|(|x|-1)}$$

Therefore

$$0 < \frac{x}{e^{x}} < \frac{x}{2^{x}} < \frac{|x|+1}{2^{|x|}} < \frac{2(|x|+1)}{|x|(|x|-1)}$$

And

$$\lim_{x \to \infty} \frac{2(|x|+1)}{|x|(|x|-1)} = \lim_{x \to \infty} \frac{2\left(1 + \frac{1}{|x|}\right)}{|x|-1} = 0$$

By the Sandwich Theorem

$$\lim_{x\to\infty}\frac{x}{e^x}=0 \quad \square$$

Lemma 3

$$\lim_{|x|\to\infty}\frac{x}{e^{x^2}}=0$$

Proof

Set x > 0

$$0 < x < x^{2}$$

$$\Leftrightarrow 0 < e^{x} < e^{x^{2}}$$

$$\Leftrightarrow 0 < \frac{1}{e^{x^{2}}} < \frac{1}{e^{x}}$$

$$\Leftrightarrow 0 < \frac{x}{e^{x^{2}}} < \frac{x}{e^{x}}$$

By using Lemma 2

$$\lim_{x \to \infty} \frac{x}{e^x} = 0 \quad \dots (1)$$

By using the Sandwich Theorem

$$\lim_{x \to \infty} \frac{x}{e^{x^2}} = 0$$

Then, set x = -t (t > 0) and thus $t \to \infty$ when $x \to -\infty$

$$\lim_{x \to -\infty} \frac{x}{e^{x^2}} = \lim_{t \to \infty} \frac{-t}{e^{(-t)^2}}$$
$$= -1 \cdot \lim_{t \to \infty} \frac{t}{e^{t^2}}$$

Also from (1)

$$\lim_{t \to \infty} \frac{t}{e^{t^2}} = 0$$

$$\therefore \lim_{x \to -\infty} \frac{x}{e^{x^2}} = -1 \cdot 0 = 0$$

Thus

$$\lim_{|x|\to\infty}\frac{x}{e^{x^2}}=0\quad \Box$$

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Lemma 4

$$\lim_{|x|\to\infty}\frac{x^2}{e^{x^2}}=0$$

Proof

Set $t = x^2$ (t > 0). Then $t \to \infty$ when $|x| \to \infty$

By using Lemma 2

$$\lim_{|x| \to \infty} \frac{x^2}{e^{x^2}} = \lim_{t \to \infty} \frac{t}{e^t} = 0 \quad \Box$$

Lemma 5

$$\forall n \in \mathbb{N} \ \lim_{|x| \to \infty} \frac{x^{2n}}{e^{x^2}} = 0$$

Proof

$$\lim_{|x|\to\infty}\frac{x^{2n}}{e^{x^2}} = \lim_{|x|\to\infty}\left(\frac{x^2}{e^{x^2}}\right)^n$$

Set
$$t = \frac{x^2}{n}$$
 ($t > 0$). Then $t \to \infty$ when $|x| \to \infty$

Thus

$$\lim_{|x|\to\infty} \left(\frac{x^2}{e^{\frac{x^2}{n}}}\right)^n = \lim_{t\to\infty} \left(\frac{nt}{e^t}\right)^n$$
$$= \lim_{t\to\infty} \left\{n^n \left(\frac{t}{e^t}\right)^n\right\}$$
$$= n^n \left(\lim_{t\to\infty} \frac{t}{e^t}\right)^n$$

By using Lemma 2

$$\lim_{t\to\infty}\frac{t}{e^t}=0$$

Therefore

$$n^{n} \left(\lim_{t \to \infty} \frac{t}{e^{t}} \right)^{n} = n^{n} \cdot 0^{n} = 0$$
$$\therefore \lim_{|x| \to \infty} \frac{x^{2n}}{e^{x^{2}}} = 0 \quad \Box$$

Lemma 6

$$\forall n \in \mathbb{N} \ \lim_{|x| \to \infty} \frac{x^{2n+1}}{e^{x^2}} = 0$$

Proof

$$\frac{x^{2n}}{e^{x^2}} < \frac{x^{2n+1}}{e^{x^2}} < \frac{x^{2(n+1)}}{e^{x^2}}$$

By using Lemma 5

$$\lim_{|x| \to \infty} \frac{x^{2n}}{e^{x^2}} = \lim_{|x| \to \infty} \frac{x^{2(n+1)}}{e^{x^2}} = 0$$

By the Sandwich Theorem

$$\lim_{|x|\to\infty}\frac{x^{2n+1}}{e^{x^2}}=0\quad \Box$$

Lemma 7

$$\forall n \in \mathbb{N}, \, \forall a \in \mathbb{R} \lim_{|x| \to \infty} \frac{ax^n}{e^{x^2}} = 0$$

Proof

By using Lemma 3, 5, and 6

$$\lim_{|x|\to\infty}\frac{x^n}{e^{x^2}}=0$$

Then

$$\lim_{|x|\to\infty}\frac{ax^n}{e^{x^2}} = a \cdot \lim_{|x|\to\infty}\frac{x^n}{e^{x^2}} = a \cdot 0 = 0 \quad \Box$$

Theorem

$$\forall n \in \mathbb{N}, \lim_{|x| \to \infty} D^n \left(e^{-x^2} \right) = 0$$

Proof

By using Lemma 1

$$D^{n}(e^{-x^{2}}) = (a_{0} + a_{1}x + a_{2}x^{2} + \dots + a_{n-1}x^{n-1} + a_{n}x^{n})e^{-x^{2}}$$
$$= \frac{a_{0} + a_{1}x + a_{2}x^{2} + \dots + a_{n-1}x^{n-1} + a_{n}x^{n}}{e^{x^{2}}}$$
$$= \frac{a_{0}}{e^{x^{2}}} + \frac{a_{1}x}{e^{x^{2}}} + \frac{a_{2}x^{2}}{e^{x^{2}}} + \dots + \frac{a_{n-1}x^{n-1}}{e^{x^{2}}} + \frac{a_{n}x^{n}}{e^{x^{2}}}$$
$$= \sum_{i=0}^{n} \frac{a_{i}x^{i}}{e^{x^{2}}}$$

Then

$$\lim_{|x|\to\infty} D^n(e^{-x^2}) = \lim_{|x|\to\infty} \sum_{i=0}^n \frac{a_i x^i}{e^{x^2}}$$
$$= \sum_{i=0}^n \left(\lim_{|x|\to\infty} \frac{a_i x^i}{e^{x^2}}\right)$$

By using Lemma 7

$$\lim_{|x|\to\infty}\frac{ax^n}{e^{x^2}}=0 \text{ where } \forall a \in \mathbb{R}$$

And the following equation holds obviously

$$\lim_{|x|\to\infty}\frac{ax^0}{e^{x^2}} = \lim_{|x|\to\infty}\frac{a}{e^{x^2}} = 0$$

Therefore

$$\sum_{i=0}^{n} \left(\lim_{|x| \to \infty} \frac{a_i x^i}{e^{x^2}} \right) = \sum_{i=0}^{n} 0$$
$$= 0$$
$$\therefore \lim_{|x| \to \infty} D^n \left(e^{-x^2} \right) = 0 \quad \Box$$

<Impressions>

It took me a lot of time to find the answers of the problem 8 and 10. That's because I had to solve two the hardest problems alone. Actually, problem 8 and 10 required the knowledges about Calculus, limit, first derivative test and such, so it was too difficult for other members of our team to newly study these contents that many Japanese 3rd grade high school students felt difficult. Eventually, I found the answers of the problem 8 and 10 by myself and I thought this class was meaningless for other members.

In addition, there were few lectures in Mathematics in our trip to Canada. Our trip was almost 6-day trip, but the mathematical lecture by Dr. Angelo Mingarelli is only once, only about 3 hours. In other time, we went boring sightseeing that had nothing to do with mathematics. The motto of this class is "Study mathematics in English". However, there were only a few hours to "Study mathematics in English" in our trip. Moreover, it was boring that Dr. Angelo Mingarelli did not ask any questions too. I didn't see the passion for mathematics in him.

Honestly, I thought this class was the waste of time for both teachers and students. I don't want to see such a boring class again.

-T. ISHIMOTO

I went Canada, it's very exciting for me. And I found a lot of interesting things at that time. So I introduce about this.

First, Canada is very big country. Load of car is too wide. Interval of houses is wider than Japan. And Canadian are less than Japanese. So, I became even more sensitive about big. Also, Canadian houses are very cute. It's colorful and beautiful. And Canadian people are very kind. When I missed, they did not get angry at all. For the above reasons, I thought I want to live the Canada. But I do not like cold place. Canada is very cold. I did not think it is the ideal places for humans live in Canada. So it may be difficult for me.

I particularly think to be interested is construction of roof. It snow a lot Canada so snow accumulates everywhere. Of course, On the roof. It is danger for people. So roof of Canadian house is sharp. I feel it is like we are inside the picture book.

I learned new things about mathematics. Our group problems are difficult for me. So I studied mathematics harder than before. By doing so, I get new knowledge of it. For example, limit value, degree polynomial, mathematical induction, and the sandwich theorem. It was hard for me but it is exciting and I like mathematics more. I think it was good to study our problem. I

will keep studying it. And Dr. Mingarelli's lecture is interesting for me. Dr. Mingarelli's lecture is about Leonardo da Vinci. He has a lot of achievements of many kinds study. He is smart but he is clumsy. For example, he has a lot of mistakes in his records. And his mistakes are very easy. It is very interesting for me. Also I like "Algebra". This idea is related to circle. I think I want to study about it more. I can't understand this perfect so I want to study it again. It was a good experience that solving math problems in English.

Finally I enjoyed being in Canada. I want to go again. So I study English more because I want to speak Canadian people more.

— M. HAMADA

In this problem, I dealt with the infinite number of powers of X. It was very difficult to handle such a huge and complicated number. However, I was able to easily represent complicated numbers with the knowledge of logarithms etc. Also, when I come up with an idea when I am worried without solving a problem, and when a problem that I thought was difficult and could not be solved was solved, I was able to get a sense of accomplishment. By solving this problem, I was able to reaffirm the fun of mathematics. Also, I think that the more difficult problem, the greater the sense of accomplishment. Because of this reason, I want to solve various mathematical problems.

On my Canadian study trip, I learned not only math but also various things. First of all, it was the first time to fly for more than 13 hours, And the first trip in the America was a different experience from Europe and Britain. I was nervous in the cabin, but I enjoyed watching the movie and looking out of the window. The flight was over more quickly than I expected. When I went outside, I felt a pain on my face in the cold that I have never experienced before. However, the unexposed parts were not too cold since the humidity is lower than in Japan. I was impressed when I got around the city on the bus and arrived at the Parliament Building because it was a beautiful building like the one in Harry Potter movies.

The next day we went around the city by bus from morning. I ate lunch at a shopping mall at that time, then I learned a lesson there. It is the difference between the size of overseas and Japanese food. In my subjectivity, I felt the medium size in Japan was like small size overseas.

The third day of the trip was a meeting at Carlton University, the main purpose of this trip. It was my first time to see a university abroad, and it was good opportunity for me to see the campus. Overseas universities are bigger than Japanese universities and I wanted to go there. There were forests, buildings and parking and I was very surprised at its scale. I had a chance to see the classroom a little, and it seemed to be at home. It seemed to be fun to take the exam with drinking Starbucks drinks. I thought I could concentrate on the exam by drinking a coffee and I want to try it.

Dr. Mingarelli gave us a lecture about Leonardo da Vinci. Leonardo da Vinci had all kinds of knowledge, including music, architecture, mathematics, geometry, anatomy, etc. and I thought he was the perfect person. However, he also made a rudimentary multiplication mistake, and the all subsequent problems were wrong. When I heard that story, I thought that even those who seem perfect make a mistake. I also make many mistakes, but I got a little confident. While we were making our presentation, the teacher nodded to listen carefully and praised. Because of that, I became confident in math and making presentations.

Japan has small land and few immigrants. On the contrary, Canada is the second largest land in the world and is full of immigrants. Because of that, there were differences in political systems and states. Also, it was very exciting to talk with local people in English as the language is different. After that, I was able to realize the difference from Japan in the sightseeing of the Parliament Building. I was surprised that the scenery would change so much just because of different cultures and places in the country.

There was also a difference in humanity from Japan. There were many gentle people, even those who did not know gave me greetings in the morning and kept the doors open. When buying maple syrup at the byword market, we met a Canadian who has a Japanese girlfriend. He could not speak Japanese fluently but he could speak Japanese, because the girlfriend is teaching Japanese to him. It was mysterious to meet someone who was learning Japanese in a distant Canada with few Japanese people. But it motivated me to learn English. On the day I can act freely. We went around various places without using maps, such as the Rideau Center and the banks of the Rideau River. Walking a land you haven't visited without a map was a very valuable experience. It was very good because there was a new place that was not found if I checked it in advance with my mobile phone. It was a monument written about Ottawa.

Throughout this trip, I was able to enhance my interest in mathematics, motivation in English, and greed for new things. In addition, I was able to experience a culture different from Japan, so I learned various things such as interest in overseas. Therefore, I wanted to make use of this experience in the future, broaden my horizons and look overseas. I would like to participate if there is such an opportunity again.

- K. YODO

(prior learning)

When I first saw problems we chose I thought I can not understand these problems because there were a lot of math signs I have ever seen. But I could understand little by little by support from team members. At first, I learned about differential calculus. It was very difficult for me because it is the field I did not learn before, so, I made efforts to understand about it. In addition I learned mathematical induction, binomial theorem and limit and so on. Finally I managed to understand these problems. I got sense of accomplishment and knew a fun of solving difficult math problems. Also I want to appreciate for my team members who taught me how to solve these problems. And I respect them because they solved by own strength and I want to get their skills by solving a lot of math problems in the future.

(Sightseeing in Canada)

We saw Northern Lights in Parliament on first day. It was very beautiful, so, I was impressed. Although being disappointed, we could not saw long time because the temperature was very low, which I have ever experienced.

We went to Canadian history museum and art museum on second day. In history museum, I could learn history of indigenous people in Canada. The area that things about it were displayed had strange atmosphere, so, I felt little scary. There were a lot of artworks that famous painters draw. The artwork I was interested was one drawn by Picasso, because I wanted to see his artworks in my life.

We observed parliament on 4th day. Interior of it was very beautiful and fancy. And staff talked about Canada's politics, so, we could learn Canada in detail. And I could know history of Canadian politics.

We ate dinner in Milestone restaurant. Before I went to Canada, I was concerned about food. But Canadian foods were delicious. In that, dishes we ate in Milestone restaurant were very delicious. Also paying money including chip by myself becomes good experience for me.

We could see Rideau canal every day. It was World Heritage Site and very wide as well as long. It was frozen, so, I expected to see people doing skating. But unfortunately we could not see them. I was surprised to hear that there are some students who go to the school by skating and children about 4 age can do skating. I want to live in such a special life.

Our group went shopping in Rideau centre and Byward market. Rideau centre is very big shopping mall, so, we could enjoy shopping. On the other side, Byward market is area that many small shops are lined up. It was very beautiful landscape. (Lesson by Dr. Angelo Mingarelli)

We took a lesson by Dr. Mingarelli in Carleton University. This university has a large area and likes college in foreign movies. The lesson was about Leonardo da Vinci. I thought that he was artist, so, I was surprised when I heard he was expert in mathematics. Dr. Mingarelli taught us his achievement of old days clearly, so, I could understand even in English. This precious experience that he taught us leads to rising our math skills and interests about mathmatics.

And we had presentation and received comments by him. I was very worried because rehearsal carried out in Japan was not good. However, after our presentation, he said that you are excellent. I was glad to hear that. And this comment leads to my confidence.

I thought that I must study math harder and harder and make efforts after I go back to Japan. I wish to tend my feeling of appreciate to Dr. Mingarelli by letter I wrote.

(Lord Elgin Hotel)

This hotel was very big and noble. I was looking forward to stay this hotel. Breakfast and dinner are very delicious and room cleaning is very good. If I went to Canada, I can not stay this expensive hotel, so, it was precious experience. Also setting chip was experience that we can not do in Japan.

(Conclusion)

I did a lot of precious experience in Canada. These experience can be used in my life in the future. And above all, this trip had a lot of fun, so, I want to come back to Canada. I want to appreciate teacher Akiyama and teacher Ono.

- M. YOSHIIKE

We started math problem research for each group in October. My group problem was difficult to this problem for me. Because it was a problem that used differential and limit value. I have never learned differential and limit value. So, I learned them before solve this problem. When I do not understand, my group mate taught me how to solve. At the same time, we studied about Canada. For example, about Ottawa city, Carleton University and Lord Elgin Hotels. My group looked into the Houses of Parliament. I learned that fire broke out at there in 1916, but miraculously the library did not burn. I bought what I needed in Canada in December. At a advance briefing, my teacher said that the temperature in Canada would be –28 degrees, so I

bought a lot of cold protection.

And finally the day that I go to Canada came. I was looking forward to this day. However, I was very tired because I had a school exam and a college class the day before. For that reason, I slept most time that I got on an airplane. We got on an airplane for 13 hours but I felt very fast. It was really cold when I arrived in Ottawa because snow and wind were blowing and the temperature was –4 degrees. First, we went to the Houses of Parliament after arrive at Ottawa. We saw projection mapping at Houses of Parliament. It was very beautiful. It was like Disney's projection mapping. And day one is over.

On the second day, we first visited Carleton University. I was surprised to see Carleton University. Because the university was very large and there were a lot of colorful places such as walls. The colorful walls looked so-called "Instabae". We took a lot of pictures there. Next, we went to a shopping mall and had lunch at the food court. We decided stores that I want to eat and ordered to myself. I realized my lack of English when I ordered food. However, the clerk listened gently a lot of times and was able to place an order. I ate something like lap sand called "pate". It was almost delicious but very large and the sauce tasted strange. After lunch we went to the museum. Most of the museum descriptions were written in French and I was not quite sure. The only thing I enjoyed was the kids' space. The kids' space had theater settings and costumes, so I enjoyed it regardless of language. The next place I visited was an art museum. And what I thought at the museum was that there were a lot of nude pictures. I was wondering if this is related to Canada. We had dinner at a restaurant called "Milestone". However, I hurt my stomach sick because the "pate" that I ate at lunchtime was not fit and I could only eat a small amount of salad. In the evening I prepared for the next day's math presentation. I was nervous because I did not know what Dr. Angelo MINGARELLI was.

On the third day, I went to Carleton University by a citizen bus. Unlike Japanese buses, Canadian buses pull strings instead of bells when getting off. When we arrived at Carleton University, we first had a lecture by Dr. Angelo MINGARELLI. It was mainly about Leonard Vinci. He seems to have lost all the calculations because he made one mistake. After the Dr. Angelo MINGARELLI lecuture, we presented by each group. Dr. Angelo MINGARELLI was very kind and praised our announcement and I was relieved. We had lunch at Carleton University. I had to spend time buying here again. However, the clerk kindly told me. Dinner was Chinese but with unique flavors and spices. I broke my stomach this night.

On the fourth day, we went to cafes and Rideau Centers. I was able to buy a lot of souvenirs. After that, we visited the Houses of Parliament. We had dinner at the hotel. The appetizer was not delicious but the beef steak was delicious.

The last day was snowy and cold. The other day the minimum temperature was -4 degrees. However, I did not felt cold because I took measures against the cold.

I have felt in math research and research trips. First, I have felt in math research. I was happy when this problem was solved. Also, I got confidence in solving mathematics. I wanted to challenge difficult problems in the future. Second, I have felt in research trips. There are a lot of gentle people. I had many people help. For example, they picked up and delivered a muffler, and taught me how to buy foods. I heard that there are bad people in Canada, but I found that some people are good. There are bad people and gentle people in Japan. Therefore, I thought that nationality was not related to personality. Also, I thought there were many people in thin clothes. I thought the average temperature was lower than in Japan, so I wondered if Canadians would not feel cold at -4 degrees. And, there were no high buildings where people lived except for hotels. There are no apartments, the house was only up to the second floor. This is very different from Japan, which has a lot of high apartments and a lot of 3 story house.

I thought this was because Canada has a larger area than Japan and a smaller population than Japan. Therefore, I think the land per capita is larger than Japan, and I do not need to make it as high as Japan. The first is money. Canada has no cents. Therefore, payments less than 5cents are rough. So, most people paid by card. I think it is because you can buy a card at the original price. Also, I went to Canada to reaffirm Japan's goodness. The first is a tip. In Canada, if you eat at a store, you pay 10 to 20% of the money to the clerk in addition to the price of the dish. It is very troublesome. Because you have to think about the amount you pay yourself. Japan thought it was easy because they did not think about tips. Second, the food is delicious. I broke my stomach twice because food was not fit. There were some dishes that I felt were not delicious. So I thought it would be happy to be able to eat something delicious in Japan. I thought I liked Japanese food and Japanese flavors. There are good and bad memories on this research trip. However, it was a good experience for me to discover new things and re-recognize them.

- M. FUJISAWA

Trisection Angle

Ryuki SEKIGUCHI, Shun SAKUMA, Sayuri TOKUMITSU Group2

Question

This problem is considered from ancient Greek era and had remained unresolved for over 2000 years. Also, this is famous as one of the three drawing problems in Greece. Three drawing problem in Greece are
[Trisection angle]
[Doubling the cube]
Draw a cube whose volume is twice that of a cube with one side length.
[Squaring the circle]

Make a square with the same area as a circle with a radius of 1.

Solution

There is a very important rule to solve these problems.

- 1. We can use ruler only to draw a line connecting two different points
- 2. We can use compass only to draw a circle with a given radius around a given point
- 3. It is banned to measure the length with ruler or compass.

Archimedes thought we can draw such angles with rulers which have 2 point on itself.

How to trisect angle by Archimedes



When you are given an angle x as shown above figure, extend the bottom line and draw a circle centered on O with radius r. (r can be any length) plot two points A and B on the ruler so that AB=r. Put the point B on a circle and move a ruler so that the point A is on the extended line. Extend AB so that it is on the intersection of the line above the corner x and the semicircle centered on O. let y be created corner. At this time, y=x/3. Then we could divide the original angle into three equal parts.

In these ways, we can draw any trisection angles but it is not solution for the problem. Because it is not allowed to use rulers to measure the length. On that way, we drew two points on the ruler. It is not allowed.

We can draw trisection angle in a interesting way. With origami!!!!

- 1. Fold origami appropriately to make any angle
- 2. Fold twice at the same interval and make a crease
- 3. Fold and crease so that A is A' on PC and C is C' on BE
- 4. If A'B' and C'B' overlap with respect to the midpoint B' of A'C', the fold passes through C
- 5. In 4, CB' and CC' give the angle halves.

<Impressions>

Traveling to Canada was the first overseas trip in my memory. My parents say that you went to Hong Kong when you were a child, but I don't really remember at all. So, even if purpose was to study math, I was looking forward to December. I understand that my main purpose is to study mathematics in English for one year in class, but actually I was most excited about going to Canada in my mind. And it was very excited to go and had a lot of exciting experiences. I write about what I experienced in Canada and how I changed through it.

First of all, the shortest way to describe my Canadian experience is to "get it for the first time". The first overseas, the first shopping in English, etc. Everything I saw and heard was fresh and exciting. What impressed me in particular was that the layout and colors of the buildings and roads were completely different from Japan. Of course, because of the size of the land, but I felt that it had an effect on people's character and behavior. In Japan, there are many people who are in a hurry even though there is nothing to harry, but in Canada, there are few such people. And things impressed me at most was that there were many different races. In Japan, I feel like a majority, but globally, the Japanese people are very minority. Just a little walk through the city and pass by a wide variety of people. For the first time, there were statements like racism. Suddenly, while walking down the street, he shouted, "Are you from North Korea ?!" I was not deeply hurt, but I was surprised anyway. As you can see, there were so many first-time experiences, good and bad, in Canada.

Through this experience, I firstly realized my smallness. I was convinced that I had to learn

more. And I am constantly trying new things everyday to try and improve myself with new experiences. I started studying French a bit before I went to Canada.

- S. SAKUMA

I went to Canada for the first time. I got a lot of knowledge and knew what I didn't know. I was really looking forward to going abroad since elementary school. I wanted to how I could communicate in English I learned in junior high school and high school. I would like to share what I learned after a week, what I thought at the end of the day, what I thought of when I presented my mathematics in Canada. On the first day, I went to the hotel. Canada was very cold because it was snow. The second day we went to Ottawa and Quebec. Our group got up early in the morning and enjoyed the morning Ottawa. I was very surprised. Because Ottawa people had coffee in the one hand in the morning and they didn't wear much. When my eyes meet their, they said hello. We decided to say hello. Many cars don't pass the street in the morning, and the air was very clean and delicious. On that day, I saw Carlton University and embassies in various countries and a museum in Quebec. Carlton University is very large, so I didn't know where I was now. The museum was very large, quiet and mysterious. I didn7t know the art work, it wasn't very interesting. Every Canadian facility is very large and my feet are very tired. But I was looking forward to dinner that day at a good restaurant. At that place, I learned a lot about overseas menus and tips for paying clerks there. The second day was very important day in Canada. I had explain the mathematics problem coming from Dr, Mingareel without using Japanese. I practiced a little at school, but I was really nervous during the actual production. Did teacher understand in the English I am speaking now? But after our presentation, teacher praised our very much. After the presentation, I was very relieved. I was very glad that there was no mistake in the same practice. I was very surprised at University. A test was being conducted in the next room we had presentation. The students were drinking during the test. I can't think of it in a Japanese exam. The local university students were very kind. He told me when I didn't know the shop was open and where it is. And he took me with me again. The university restaurant was very large and had many menus. After college, it was really fun. We went Byward market. There were various things sold and very cheap. The beavertail is very delicious. I was very happy. Because I bought very much. The third day was a free all day. Our group went to a big shopping mall and Byward market. The shopping mall had Disney store which my favorite place. There were a lot of things that limited to overseas, cute clothes, toys,

etc. There was a New York Fly which I really wanted to eat. On that day I could buy lots of things and talk a lot with local people. Then I knew that how much I could speak English. The local people understood me as a tourist and spoke English slowly. It was very easy to hear and understand. But when I talked, the words didn't come out and I got very poor English. I knew I couldn't speak English yet. I didn't want to go home very much on the last day. For the first time in Canada, I was very happy and I know that speaking English is very fun. I think that this trip to Canada is very fun and a very important memory in my life. This time I was able to preach and explain mathematics in English. Next time I have a chance to go abroad, I would like to make use of what I learned this time and interact with a lot of local people and have more fun.

- R. SEKIGUCHI

When I choose this class, I was little afraid of all. Because I am not good at mathematics and English.

However, I could understand gradually. By researching myself or asked to someone. I realized we can do what we are not good at by effort.

In Canada, we conduct a presentation at Carleton University for Dr. Mingarelli. I got nerves last night. But, in the presentation, he praise us a lot. And my proof by using origami was said so good. It was incredible. I surprised and feel happy. Although study was hard, this class very rewarding.

I learned not only mathematics but also the importance of English. In Canada, I could talk to someone in English a lot. I had courage of talking to people. I could talk much before. I realized again and again that if I can speak English, I can talk with people.

I learn by this class, we can do many thing by effort and courage.

— S. TOKUMITSU

Why are hot things red? Ryota OCHIHATA, Asahi SAKA, Ann NAKAJIMA Group 3

Generally speaking, the most people have the image that something hot is red. It is not sensation. This principle has been elucidated mathematically. It is related to the frequency of radiation emitted from heated body. And, there is a Wien's displacement law used for elucidating the principle. It's based on temperature and Planck curve. The Planck curve expresses the relationship of the radiant energy of a black body and its wavelength. A black body is an object that can radiate heat as an electromagnetic wave. At first glance, it looks difficult. But we will solve the problem by using two basic solutions. So let me get into a main part.

Solution

I. Algebra Problem

In this solution, we find the constant **b** that can show table 1. Therefore, when I take the numerical value of table 1 to a graph, a graph such as figure 1 is done. set λ = Peak Wavelength and T = Temperature. When I read it about curve of a done graph, an expression of the inverse function $\lambda = \frac{b}{T}$ (1) comes out. When I substitute numerical value of table 1 for an expression (1), we can find a value of constant **b** is approximately 2,898. From the above, we can express the expression that expressed temperature and frequency with $\lambda = \frac{2898}{T}$.

II. Calculus Problem

Set $\alpha = 14394$

From the assumption, the relationship between temperature and frequency can be expressed as

$$I(\lambda, T) = \frac{A}{\lambda^5 \left(e^{\frac{\alpha}{\lambda T}} - 1\right)} \text{ on the figurer } \textcircled{0}$$

$$U = \frac{A}{\lambda^5} \xrightarrow{dU} (d) = U' = -5\frac{A}{\lambda^6}$$

$$V = e - 1 \xrightarrow{differentiate} \frac{dV}{d\lambda} (=V') = e^{\frac{\alpha}{\lambda T}} \left(-\frac{\alpha}{\lambda^2 T}\right) = -\frac{\alpha}{\lambda^2 T} e^{\frac{\alpha}{\lambda T}}$$

Then use the quotient rule, ...

$$\frac{d}{d\lambda} \left(\frac{U}{V}\right) = \frac{U'V - UV'}{V^2} = \frac{U'}{V} - \frac{U}{V^2}V' = \frac{U'}{V} - \frac{U}{V^2} \cdot \frac{dV}{d\lambda} \cdots \square$$

$$\textcircled{D} \times V$$

$$U' - \frac{U}{V} \cdot \frac{dV}{d\lambda} = \frac{dU}{d\lambda} - \frac{U}{V} \cdot \frac{dU}{d\lambda} \cdots \textcircled{D}$$

When (2) is 0, it takes an extreme value (maximum or minimum).

$$\begin{aligned} \frac{dV}{d\lambda} &- \frac{U}{V} \cdot \frac{dV}{d\lambda} = 0 \\ &- \frac{5A}{\lambda^6} - \frac{U}{V} \Big(-\frac{\alpha}{\lambda^2 T} e^{\frac{\alpha}{\lambda T}} \Big) \\ &= \frac{5A}{\lambda^6} - \frac{A}{\lambda^5} \cdot \frac{1}{e^{\chi} - 1} \cdot \Big(-\frac{1}{\lambda} \chi e^{\chi} \Big) \\ &= -\frac{5A}{\lambda^6} + \frac{A}{\lambda^6} \cdot \frac{\chi e^{\chi}}{e^{\chi} - 1} = \frac{A}{\lambda^6} \Big(-5 + \frac{\chi e^{\chi}}{e^{\chi} - 1} \Big) \\ &= \frac{A}{\lambda^6} \Big\{ \frac{-5(e^{\chi} - 1) + \chi e^{\chi}}{e^{\chi} - 1} \Big\} \end{aligned}$$

From $\frac{A}{\lambda^6} \neq 0, e^{\chi} - 1 \neq 0$

 $-5(e^{\chi} - 1) + \chi e^{\chi} = 0$ $-5(e^{\chi} - 1) + \chi e^{\chi} = -5e^{\chi} + 5 + \chi e^{\chi} = 0$

So, $5e^{\chi}-5-\chi e^{\chi}=0\cdots$ (3)

Sine this formula ③ cannot be solved by hand calculation, we substitute consecutive number to obtain an approximately value, and find the number that is closest to 0. Table 2 shows approximate values. When the value of X appoaches 4.965, the formula ③ appoaches 0. How ever, when the value of X reaches 4.966, the formula ③ becomes less than 0. From this, assuming X = 4.965, and substituting it into formula ③, it becomes $4.965 = \frac{14394}{\lambda T}$. When I transform, it becomes $\lambda = \frac{2899}{T}$. From the above, the formula in table 2 can be expressed as $\lambda = \frac{2899}{T}$.

III. Conclusion

Since the constants of the formula obtained in I and II are approximate values, we can see that the Wien's Displacement Law holds good in here. From this, it can be said that hot things are red.

Temperature	Peak
(K)	Wavelength
	(microns)
10,000	0.2898
9,000	0.322
8,000	0.362
7,000	0.414
6,000	0.483
5,000	0.579
4,000	0.724
3,000	0.966
2,000	1.449
1,000	2.828
500	5.796
300	9.660

4.5 4.9

4.95 4.96

	9.660	_	
Table	: 1		
	Eq. 3 5.873127 17.16717		
	49.59815 40.00857		atensity 1 (arb. units)
	17.16717 35.17107 49.59815 40.00857 8.428978		

2.058748

0.703752





<Impressions>

I had a very precious experience in Canada. It was the best memory of high school life. It was so fun. Those were a very memorable 6 days. I have never been to Canada before. So I was really looking forward to going. Traveling by airplane was long and narrow. So I was very tired. But when I arrived there, I was surprised at differences from Japan. There were many castles and churches. Those were very beautiful. Also there here were Christmas trees everywhere. I was excited. And it was very cold. I really enjoyed sightseeing. I was able to visit various places freely in Canada. For example, Rideau Center, Byward Market and so on. I want to go to Ottawa again and visit many other places. Also, I would like to visit other cities in Canada. It was a very good experience to be shopping alone. I've been studying English until now, but I couldn't tell my mind to local people well in English. So, I will try hard to improve my English skill. In particular, I want to do my best to improve pronunciation. Among my memories in Canada, Carlton University was the most impressive. I went to an overseas university for the first time. That was very interesting. Carlton students looked fun and attractive. There were people of various nationalities. I thought that if I enter a foreign university, I can make many friends whose nationality are different from me. It looks very fun. Dr. Mingarelli's class was an impressive class that I had never received before. He was very gentle. And his class was very very interesting. I became to be interested in Leonardo da Vinci. I had a strong image that Leonardo da Vinci was one of the most famous painters. But I learned that he also had a great influence on the world as a mathematician. I often make miscalculations, but Leonardo da Vinci also made many miscalculations too. He made an effort and became a one of the most important mathematicians in the world. So, I want to study harder like him to be good at mathematics. Presentation to Dr. Mingarelli was also good. We tried very hard to solve an our problem. The problem we committed is "Why are hot things red?". I was not good at mathematics. So, I thought it would be very difficult and impossible to solve when I saw this problem first. In particular, the calculus part was the most difficult. It was hard to make the formula as simple and easy as possible. Also, it was the most difficult to make a presentation manuscript in a concise manner too. However, we were able to solve it with the efforts of all the team members in the end. I felt a sense of accomplishment. I was happy to see Dr. Mingarelli's smile when we were presenting. After the presentation, he said to our team "Very good!" This word made me very glad. The presentations from the other teams were also very interesting. Among them, the explanation using origami was impressed me the best. It was easy to understand. Our team will also consider more innovative explanations such as that explanation. I thank Dr. Mingarelli for hearing the our presentation. I want to tell him "I will try hard".

Through studying at Kyōyōsōgō for a year, I realized how difficult it was to teach people easily. We had a hard time to execute this. Even if we understood well, it was difficult to teach it easily to others. The poster and manuscripts have been devised to concisely convey by us. We also judged whether it was really necessary explanation. We did everything we could. However, following the advices from the teachers, we found something to improve. I would like to make improvements for the school presentation and the Kanto Games so that everyone can be more interested in our study. And I would like to learn more and make better presentations.

In next year, I will be in the humanities class. So, there is less opportunity to learn mathematics. However, I am going to learn math little by little by making use of what I learned this year. And some day I want to be able to prove what I am interested by using my mathematics knowledge. I will do my best.

— A. NAKAJIMA

I'll tell you something about this problem study and trip. In the first place, the reason why did I go to Ottawa is a lottery. Firstly, I wanted to go to Poland, but I was excluded.

Then, I deal with a second lottery. So I decided to go to Ottawa. The beginning, I wondered that "How does this subject turn out?". To study math in English is my first experience, so I felt it to be uneasy. While I push forward a class, I met many different words I didn't know. But I tried it hard to understand these words. So I could understanding almost of this subject. Next, I'll tell you the reason that I chose this problem. Firstly, we were shown some problems from Mr. Akiyama. We thought this problem is more interesting than any other problem. So we decided to research this problem. We couldn't do anything for this problem when we saw at the first time. It's because, to solve this problem, we had to use differential calculus. We have ever studied it. So we could not go well. Also there is a time we were troubled to change the problem.

But there is no other problem as change, we continued to solve the problem. We stayed at the school in the afterschool, we talked with Mr. Akiyama, we found the key to solution. Then, the thing which I felt fuzzy in among oneself cleared at a stretch.

And it is a day of the departure at last. As I heard that Ottawa was very cold, I dressed warmly and boarded an airplane. I spent a time with seeing the movie, reading a book in the plane. It was so comfortable. Then we reached in Ottawa. As expected, Ottawa was very cold like as we can't feel in Japan. Through this trip, I was able to have a lot of experiences in Ottawa. Also, I was able to understand that there are somethings that we do not know in the world being infinite. I thought to untie it one by one is so interesting. I wan to appreciate that I act together to an active member very much. Thank you.

— A. SAKA

At first, I will talk about why I choose this class before my impression. I wanted to go to foreign country where English is the native tongue. I have traveled overseas several times. But, I couldn't speak English at those times. I could go to another class that learns English, but I it was unsatisfactory to learn only English. Since I attend the class one year, I will study English and math as well. So, I decided join this class.

Next, impression of this class. In this class, I have been given a huge amount of homework many times when the first term. It's a math problem written in English and translation of English mathematical terms. Also, There is a homework for summer vacation. It was hard to do my homework, especially when I had a presentation. I just have to make some time. Each weekly class had 2 hours, and of course I was taking math classes in English. So, It was not very fun. Second term became, I thought I would be more busy. Certainly, I had weekly class.

But, The contents of lesson became a group work solving a problem sent by a Canadian teacher, The teacher gave us many time to solve it. Our group choose one problem" why are hot things red". It was too difficult for me. Because we had to use the calculus we haven't learned yet. Over the course of many week, we challenged a problem. Sometimes with the help of a teacher. And we did a presentation in Japan. I went to Canada with some modifications. This trip to was the my first travel to Canada. But I have been USA three times. At those travel, I was 5,6 and 10 years old. So, I didn't speak English. But this time, I've already been 17. And I learned English. I didn't think I could actually communicate with Canadian people. Nevertheless I was looking forward to this trip. The teacher said that this trip was a study trip, but when it ended, it was a fun trip like a sightseeing trip. We visited museums and art galleries. I went to Canadian markets and shopping malls and bought souvenirs. What I'm glad to see personally is road signs written by English and French. I was learning about Quebec and Canada's history in geography classes, in those class. Teacher told that English and French were official languages in Canada. So, I saw it and realized that I am in Canada. I will write about a math class I took in Canada.

trip. The atmosphere of college was very good and the teacher was very kind. About 1 hour, a Canadian teacher taught us about da Vinci. I did not know that Da Vinci did mathematics, and the class was a bit difficult, but I could understand the contents to some extent. Canadian teacher asked a few questions during the presentation, but I was relieved that it was good without any correction.

— R. OCHIHATA

Concentric ellipses and tangent line

Maaya SUZUKI and Miki YAGISHITA

Group4

Question

Two concentric ellipses are given by

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$
 (E1) and $\frac{x^2}{16} + \frac{y^2}{25} = 1$ (E2)

Find the points on E2 obtained by the intersection of the tangent line to

the point $\left(1, \frac{3\sqrt{3}}{2}\right)$ on E1.

Solution

From problem sentence and $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and $\frac{x^2}{16} + \frac{y^2}{25} = 1$ are elliptic equation, so we can draw a diagram.

First find the slope at this point for tangent line by differentiating.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$
 differentiate about both sides x



- 1. Differentiate $\frac{x^2}{4} \rightarrow \frac{x}{2} (y = ax^n \rightarrow y' = anx^{n-1})$
- 2. Use chain rule of differentiation
 - (1) Differentiate $y \operatorname{as} x$ (2) Multiply (1) by y'(3) We have (1) $\frac{y^2}{9} \rightarrow \frac{1}{9} \cdot 2 \cdot y = \frac{2}{9}y$ (2) $\frac{2}{9}y \cdot y' = \frac{2}{9}yy'$

3. ① is the real number, so ① became 0 by differentiating. Therefore,

$$\frac{x}{2} + \frac{2}{9}yy' = 0$$
$$\frac{2}{9}yy' = -\frac{x}{2}$$
$$yy' = -\frac{x}{2} \cdot \frac{9}{2}$$
$$y' = -\frac{9x}{4y}$$

This equation is derivative.

Then substitute the point $\left(1, \frac{3\sqrt{3}}{2}\right)$ for y' because we must find the slop of tangent line at $\left(1, \frac{3\sqrt{3}}{2}\right)$

$$y' = -\frac{9 \cdot 1}{4 \cdot \frac{3\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$$

So the slope is $-\frac{\sqrt{3}}{2}$, therefore $y = -\frac{\sqrt{3}}{2}x + a$ and through the point $\left(1, \frac{3\sqrt{3}}{2}\right)$ to find a

$$\frac{3\sqrt{3}}{2} = -\frac{\sqrt{3}\cdot 1}{2} + 1$$
$$a = 2\sqrt{3}$$

Then, solve the simultaneous equation

$$\begin{cases} y = -\frac{\sqrt{3}}{2}x + 2\sqrt{3} \cdots \\ \frac{x^2}{16} + \frac{y^2}{25} = 1 & \cdots & 2 \end{cases}$$

Substitute (1) to (2)

$$\frac{x^2}{16} + \frac{1}{25} \left(-\frac{\sqrt{3}}{2}x + 2\sqrt{3} \right)^2 = 1$$

$$25x^2 + 16 \left(\frac{3}{4}x^2 - 6x + 12 \right) = 400$$

$$25x^2 + 12x^2 - 96x + 192 = 400$$

$$37x^2 - 96x - 208 = 0$$

$$x = \frac{48 \pm \sqrt{2304 + 7696}}{37}$$

$$x = 4, -\frac{5}{37}$$

The problem of using square completion

Sodai KOIKE

Group 4

Question

Find the largest value of xy that x + y = 4 without Calculus.

Solution

Move *x* of x + y = 4 to the right

$$y = -x + 4 \cdots$$

Assign (1) to xy

$$xy = x(-x+4)$$

= $-x^{2} + 4x$
= $-(x^{2} - 4x)$
= $-\{(x-2)^{2} - 4\}$
= $-(x-2)^{2} + 4$

Draw a graph of $-(x-2)^2 + 4 \ (0 \le x \le 4)$



From the graph, the vertex of this graph is (2, 4) So, when x = 2 and y = 2, the largest value of xy is 4

On the other hand,

If we use Calculus,

$$f(x) = -x^2 + 4x \cdots 2$$

Differentiate this function,

$$f'(x) = -2x + 4$$

$$x$$
 \cdots 2 \cdots f' $+$ 0 f \nearrow 4 \searrow

Therefore, the expression of (2) is an upwardly convex graph with x = 2 as an axis. So, when x = 2 and y = 2, the largest value of xy is 4.

Yuki TAKAHASHI

Question

Two wides of radius 8cm and 2cm are tangent to each other. Find the length of their common external tangent.

Solution



This radius is 8cm This radius is 2cm

How to solve this problem.

First, measure the distance center of circle A and center of circle B.

And that distance is 10cm.

By this equation, can solve to use 10cm.

$$8 - 2 = 6$$

So,OP' = 6 cm
K1K2 = $\sqrt{100} - 36$
= $\sqrt{64}$
= 8 c m

Therefore, K1K2 is 8cm.

<Impression>

I went to Canada to study mathematics and sight. I have never been to a foreign country before, so I learned a lot of things through this study trip.

The first day morning, I was busy packing because I had exam the previous day. Fortunately, preparation was in time because the flight was in the evening. I arrived at Haneda airport and had a lunch with one of my friends. Then 14 hours flight including transit. I was very scared to take the plane because I was 12 years old when I last got on the plane and I don't like shaking like a roller coaster. However, It's good for me to watch many movies on the plane because I like a watching movies. Then I arrived at Toronto and I had to pass immigration check, however I wasn't able to pass and taken to other room. I didn't understand why I was stopped at that time and even now. I managed to pass with the help of a tour conductor. I was scared but it was a good experience now. I got on a domestic flight and at last arrived in Ottawa. I got on a chartered bus and went to National parliament building and saw its project mapping. It was very cold however the landscape was very beautiful and I was impressed. After that I arrived at Lord Elgin Hotel and slept.

The Second day, first of all it was very surprised by morning breakfast. The type of viking was very luxurious and very delicious. This day toured the same place with everyone in the charterd bus. We went to Carlton University. Then we went to the mathematics faculty of the university. I was worried about the next day's presentation because our presentation wasn't have enough time to prepare. Then we went to the shopping mall and had a lunch at Tim Horton. In Canada, Tim Horton is very famous as well as Starbucks and Tim Horton is price less than Starbucks.

Then we went to the history museum and the art museum. In the history museum, I enjoyed in the kids space. The kids space was very colorful and cute, so I took a lot of photo. I was very happy because these photo were really photogenic. I went to a milestone for dinner and ordered chicken. To tell the truth, the odor of goat cheese was too strong, so I didn't like this one. However, it was good for me to learn of the paying of chips.

The third day, we went to Carlton University again by OC Transport. The bus driver was gentle because she said "You can not need to pay a fare". First of lessons, Dr Mingarelli got a lecture which is about Leonardo Da Vinci to us. This lecture was very interesting. Next was our presentation. Our team's presentation was the last, so I was so nervous. As a result, it is good so I was very happy. We use School Cafeteria and I ate a piece of pizza which is so big. After that, we went back and went to CF Rideau Center. I had a good time to do shopping. This day's
dinner was Chinese dishes. The only sad thing was the taste of Mabo tofu was not enough spicy, however others were very delicious.

The forth day was free time for each group and our group went to supermarket named Dollarama and bought for souvenirs. Then we went to Byward market which often use to do shopping by inhabitants and bought maple syrup. You know, Canada is famous for maple syrup, so there are many kinds of maple syrup and I bought standard one. We ate Beaver tail which uses the tail of a beaver as a motif as a lunch. We ordered Cinnamon flavor with lemon together and shared with one of my friends. The taste is so delicious. Then we went to CF Rideau Center again and bought cosmetics at shoppers and MAC. I was very excited and enjoyed shopping. This day was one of my friends' birthday and we had a surprise party. I had a little time, however she looked happy, so I was very happy.

The fifth day is only to return Japan.

I was very frustrated because I can't speak English well through this study trip. This reason is so simple, however so simple is that's why I was very frustrated.

After returning to Japan, We had the poster session of SSH. I was so nervous because we didn't have enough time to prepare and other high schools were so at a high level. The result as I was expected for that people didn't see even our poster. I thought that's because was too difficult to understand mathematics in English and we should have done more catchy of poster's design.

I chose this lecture one year ago, to tell the truth I didn't like mathematics because I thought mathematics was too difficult for me, so I got this lecture to improve this bad impression. And one year later since at that time, I say clearly that mathematics is not only easy however I realized that after I solved the difficult problems I felt enjoying sometimes. So, I want to do my best without forgetting this feeling.

- M. SUZUKI

In the beginning I wanted to go to Australia, but I was defeated by lottery. At that time I felt down for the result, but now I think it was a good result. In the class I did a lot of homework during my vacation, but it was very helpful in solving the problem. Our problem was concentric ellipses and tangent line but at first we chose a different one that was very difficult so we changed the problem. We were taught by Mr. Awazawa. At the time of the presentation at Carlton University, I was pleased to be told wonderful by Dr. Angelo Mingarelli.

1st day

I took a plane for 14 hours and watched a lot of movies with almost no sleep.

Arrived at Ottawa, It was not colder than expected. the illuminations were very beautiful at the Parliament building where We visit first. Even though it was freezing cold, there was a burning fire object and I wondered this.

2nd day

In the morning I was surprised that there was no salad at breakfast.

I was so big and surprised when I went to college. The underground tunnel was so wide that I thought I would get lost, also there are a lot of paintings on the wall.

Then we went to the Canadian Museum of History. There was a place for kids and that was fun. There are houses from various countries and have entered.

At the museum, the photos took in Japan exhibition was interesting. The museum was a very beautiful building and the interior design was good.

At dinner we went to the restaurant Milestones. I ate back lib steak. It was about \$ 31 and it was really big size, taste was really delicious.

3rd day

Taking the municipal bus we went to Carlton University and took classes from Dr. Mingarelli. He said he was interested in Leonardo DaVinci and show us Homo Vitruvianus and mistakes DaVinci do. Listening to friends' presentation Dr. mingarelli asked a few questions.

Our turn was last. At first two people did it, then me and Ms. Suzuki do a presentation. I felt a little nervous. We did it well, in the way of one person writing diagrams and equations and the other reading sentences. After that We went to the food court in Carleton University and drank strawberry flavored Fanta. There were many shops. There was also a Japanese restaurant called bento sushi. Friend ate this. The cashier was a unfriendly.

When we take the municipal bus to the Byward market, there were so crowded also rough driving and I felt sick.

At the Byward market, we thought that different from our imagination. We imagined a market with a tent-like place, but it was just a line of shops. I bought macaroons and two battles of maple syrup and small elephant figurine as souvenir for grandma. After that, I went to the Notre Dame Cathedral and took a pictures.

After that we returned to the hotel and we went to a Chinese restaurant. Shrimp and sour pork

were very delicious. I was so tired that day and went back to the hotel and slept without taking a shower.

4th day

I called my friend in Japan in this morning, ate some breakfast for a while at 9:30, and went to the Rideau center around 11 o'clock.

I bought cosmetics, sweets and clothes as souvenirs at the Rideau Center. There were many shops and it was very fun.

After that, I went to a place to have a senator meeting. It used to be a station building in the past and it still remains a little. There was a Christmas tree and it was very beautiful. There was also a guide, and the tour conductor translated us so I knew it well. It was so beautiful that I didn't think it was there for a long time. After that, I went to the Rideau Center again and ate steak at the hotel.

I could see the illuminations before returning to the hotel.

5th day

We had breakfast and we went outside and it was a bit snowy! After that I took a bus to the airport. First, I arrived in Toronto, I went shopping in a duty-free shop in Toronto, and Mr. Akiyama brought me sushi. After a while, we got on the plane. There was a vegan man next to me I was a little worried because he was not eating anything on the plane. I watched a lot of movies again. When I arrived in Japan, my grandmother, mother and sister came to pick me up. I stopped at a restaurant halfway and we arrived at home around 9 o'clock.

What I thought in Canada is that there are many things. One of which is that the people of Canada are very kind. When I accidentally dropped something on the bus, she helped me find it gently, and even I couldn't speak much English, she was kind. The second is that the clerk is free. The clerk uses a smartphone even while working. That's something that's not really allowed in Japan. The third is that the smell of marijuana is occasional on the roadside. That made me feel a little sick. Fourth, there are two official languages in Canada, and I thought it was amazing that many could use both English and French. I thought it would be advantageous if I was Canadian. There were usually signs in English and French, but I was surprised that the Quebec region only needed French. I was also surprised that traffic manners and rules differed between states. Fifth, there are only about half of Canadian toilet doors. I was embarrassed to

see my feet.

Through Canada, I learned a lot and went abroad away from my parents, so I think I was a little independent. I went to Canada and had a really good experience. Thank you for your tour conductor, teachers and friends.

- M YAGISHITA

I experienced a lot on this Canadian research trip because it was my first overseas trip. I had the image of Canada being a quiet city with a lot of maple trees. Also I heard from a friend that Canadians are not very kind. I was a little worried before going on a trip.

The day before. I was preparing for my Canadian study trip. First I went to buy a lot of cold protections because Mr. Akiyama and JTB said Canada was very cold and needed lots of cold protections. For example, I bought gloves, ear covers, underwear, and down jackets. The most expensive of these was snow boots. The boots cost about 20,000 yen. I put what I bought and needed in a carry bag. And I promised to go to the airport with my friends tomorrow. I was looking forward to going to Canada tomorrow and couldn't sleep immediately.

First day. I got up and did a final check of the contents of the carry bag and rucksack. My mother sent me a car at Kichijoji because I promised my friend at Kichijoji. I met a friend and we got on a bus to Haneda. We arrived at Haneda and went to the meeting place Z, but there was nobody because we mistakenly thought that the meeting time was one hour earlier. We went to the observation deck to spend time until the meeting time. After gathering, We completed luggage inspection and immigration control. I bought water and sweets and waited for the flight departure time. I was looking forward to the first overseas airplane and I wanted to read books on an airplane but I didn't read because I could watch a lot of movies and listen to music in my seat. I saw two SF movies and a thriller with English subtitles. The food was different from the image but it was delicious. However, the cup ramen that came out on the way was not so delicious. I hurt my body because I sat for 11 hours. I arrived in Toronto. I was stopped by an immigration officer and asked to show me the contents of a rucksack. The Cairo in the rucksack was taken away by the examiner and abandoned. After that I got on a plane to Ottawa. The next seat was a foreign woman sitting and I could speak a simple greeting. It was cold when I arrived at Ottawa Airport and went outside. We took the bus and headed to the hotel. When I arrived at the hotel, I slept immediately because I was tired.

Second day. I had my first breakfast. Berry yogurt was delicious. We saw the city of Ottawa by bus and went to Carlton University. We went to Rideau Center and had lunch. The meat was delicious so I had too much to eat. Next, we went to the history museum. Old people were eating there. I think the Canadian History Museum was higher quality than the Japanese History Museum. Then I went to an art museum in Ottawa. I was surprised that there was a picture of the person who learned in the world history class. I arrived at the hotel and went to the Milestone Hotel. We ate dinner there. I ordered a steak. There are a lot of fried vegetables so I eat with my friends. After eating, I went to a shopping mall and bought sweets and water. Finally we arrived at the hotel and I went to the shower. I didn't like the hotel shampoo because it didn't foam well. And we played games and talked with friends for a long time. So I slept late.

Third day. I went to Carlton University after eating the same breakfast. First I heard about Da Vinci from a teacher at Carlton University. And we explained how to solve our problem. I was nervous but I was glad he said "you were very good". I ate pizza for lunch at college. It was delicious but quite big. After that we went to the ByWard Market. People were selling a lot of things on the road. For example, maple syrup, muffler, Christmas goods, etc. I bought maple syrup and sweets for my family and friends there. Also, I bought and drank tapioca with a friend at a nearby store. It was very delicious. After returning to the hotel and sleeping a little, we went to a Chinese restaurant called Palais Imperial. I didn't like the Chinese food that came out there. After that I went to the hotel and played games again with friends.

4th day. I had the same breakfast again. After resting at the hotel, I went to the shopping mall at 11:00. We went to our favorite store and bought a lot of souvenirs. I bought a doll and a maple cookie. For lunch we had a hamburger at the food court. I told clerk my name, but the clerk misspelled my name spell. Then we went to the parliament building. The security was strict because it was the actually used Parliament Building. A guide explained the Parliament Building. The explanation was translated into Japanese by a JTB person, so I could understand the content. Then we had dinner at Lord Elgin Hotel. The steaks and desserts were good but the others were not so good. And I spent the last night at the hotel after playing games with friends as usual.

5th day. It snowed that morning. We had breakfast and immediately put our luggage in the bus and went to the airport. I arrived in Toronto and bought a Canadian key chain. I bought sweets and water and took a plane. On the way back I saw two Japanese films. And read the assignment book.

6th day. I arrived in Japan that evening. I had dinner at the airport shortly after immigration

because I wanted to eat Japanese food right away. I ate katsudon. I thought Japanese food was better than Canadian food. After that, I went to Musashi Koganei Station by bus and returned home.

I realized that listening skills were more important than reading and speaking skills on this Canadian research trip. I want to improve my listening skills from this experience. I wanted to go abroad again with my advanced listening skills.

— S. KOIKE

We went to Canada. I will talk about my impression in Canada. Dayl, we gathered at Haneda airport. I was very nervous. Because The last time I went abroad when I was three old. But, I was comfortable on the plain. When we arrived at Canada. I felt so cold and surprised. After we arrived at load Elgin hotel. I am surprised by load elgin hotel. In the first time, I stayed at rich Hotel. I was single room. Day2, we visited in Ottawa city with Japanese guide. First we went to Carleton university. I bought Ravens T-shirt and Parker. In the first time, I bought at abroad cash register. At the history museum. I can't read explanatory text. We went milestones. I had sirloin steak. It is very delicious. Day3, we went Carleton university again. We met Dr. Angelo Mingarelli. This day is the most nervous in this study trip. I feel precious time. So, in my opinion that experience is useful in the future. If I have this time again, I want to improve moreover. Also next meeting time. I will talk about Dr. Mingarelli in English. We have lunch in Carleton university. Many kinds of shop in university. For example pizza shop, Japanese food shop, etc... I was surprised at menu. Afternoon we went to ByWard market. I bought a lot of souvenirs. For example maple syrup, maple tea, maple butter and maple cookies. My friends were pleased. Moreover I ate beaver tale and tapioka. These are very delicious. I want to eat again.

At dinner we ate Palais Imperial. It is Chinese food. I think that this is not delicious very much.

My thinking, it maybe is bullied Canadian by chef. But I can have them. Day4, we have free time. I waited for this time. First we went to Riedau center. In that place, we bought variety goods. These are almost souvenir. Next we walked around hotel in Ottawa city. I felt very cold. Afternoon we could went houses of parliament. There is so slight, and nervous. Especially baggage inspection. Guard man may Strong. Because they have a lot of muscle. I felt super fear. In houses of parliament, it is very wide. It is similar to Japan. We have Canadian guide that time. I can't listen very much. I wanted to listen More over. At dinner, we went to load elgin hotel table. First we had salad and soup. I can't eat this salad. It is not delicious by me. But soup is very delicious. Next foods is steak. It is very delicious in Canada. I wanted to have more this. Next dessert, we had cream Brulee. It is so amazing. At that time birthday boy and birthday girl are celebrated By load elgin stuff. I felt happy. I confirmed load elgin's good point. These stuff are so super Correspondence. It is perfect. Day5, it is last day. We left at load elgin. I felt disappointed. I wanted to stay more over. We went to Ottawa airport. So we went to Toronto. Next Haneda. I am so tired. So I fall asleep. When I arrived at Japan. I felt a little warm. I was experienced variety. For example, OC Transpo and system of chip. This experience is useful in the future. I want to go canada. In that time, I want to go any other place. If I have chance to go canada. I want to explain to Dr. Mingarelli moreover.

—Y. TAKAHASHI

The Close Encounter to the Sun of Barnard's Star

Shion KOMINE, Rina NIKAIDO,

Seigo TAKIKAWA, Manato KOMAKI

Group5

Question

Stars travel along different orbits through the Milky Way. Near our sun, stars are going

mostly in the same direction, but from time to time they pass close together. Barnard's Star is currently in the constellation Ophiuchus, but travels across the sky so quickly that it traverses the diameter of the full moon every 180 years.

With the sun at the center of a Cartesian coordinate grid, Barnard's Star can be represented as a point located at (2.0, 5.6) where the units are in light years. But because it is moving through space at a speed of 143 km/sec, its future position relative to the sun changes quickly in time. The parametric equations for the X and Y location of Barnard's Star can be approximated as follows:

X(T) = 2.0 + 0.09 T Y(T) = 5.67 - 0.25 T

where T is in thousands of years from the present time, and all units are in light years.

Problem 1

What is the distance to Barnard's Star at the present time?

Solution

We know that the sun is at (0,0) and Barnard's Star is at (2.0,5.6) So, by using Pythagorean's Triples, the distance between Sun and Barnard's Star is

$$d = \sqrt{(2.0^{2} + 5.6^{2})}$$
$$= \sqrt{35.36} \approx 6.0$$

So, 6.0 light years

Problem 2

What is the equation of the line y = Mx + B that is represented by the parametric functions?

Solution

From examination sentence, we delete T

$$X(T) = 2.0 + 0.09T$$

Y(T) = 5.67 - 0.25T

We set X(T) = x, Y(T) = y

-200-

$$x = 2.0 + 0.09T$$
$$T = \frac{x - 2.0}{0.09}$$
$$y = 5.67 - \frac{0.25(x - 2.0)}{0.09}$$

$$=-2.78x+11.23$$

We round off to the first decimal place.



The formula of the distance between point and line



We set the line *l*: ax + by + c = 0 and the point $A(x_0, y_0) \quad H(X, Y)$ Then, normal vector of *l* is (a, b) and \overrightarrow{AH} is parallel to normal vector of *l* So, $(X - x_0, Y - y_0) = t(a, b) \quad t \in \mathbb{R}$ ______1 *H* is a point on the line *l* so, aX + bY = -c _____2 Taking the inner product of (a, b) and the equation 1 $a(X - x_0) + b(Y - y_0) = ta \times a + tb \times b$ _____3 -201From 23

$$-c - ax_0 - by_0 = t(a^2 + b^2)$$

 $a \neq 0$ or $b \neq 0$ so, $a^2 + b^2 \neq 0$

$$t = -\frac{ax_0 + by_0 + c}{a^2 + b^2}$$

Therefore, the length of AH that is the length of t(a, b) is

$$d = |t|\sqrt{a^2 + b^2} = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

Problem 3

At what time, T will Barnard's Star be closest to the sun along this trajectory?

Solution

From problem 3 answer,

$$2.78x + y - 11.23 = 0$$

use the formula of the distance between point and line,

$$d = \frac{|11.23|}{\sqrt{(2.78)^2 + (1)^2}}$$
$$= 3.8$$

So, 3.8 light years

Also, use the Pythagorean theorem

$$(3.8)^2 = (2 + 0.09T)^2 + (5.67 - 0.25T)^2$$

Differentiating the formula with respect to T

$$0 = 2 \times (0.0081T + 0.0625T) - 2.475$$
$$0 = 0.0706T - 1.24$$

Therefore,

$$T = \frac{1.24}{0.0706} \approx \underline{17.6}$$

<Impressions>

I was so happy to participate this mathematics trip. I saw a lot of interesting things and got many lessons.

When I first heard the presentation of mathematics in English last year, of course I couldn't make sence what they said at all. However everyone on the stage looks really cool then I wanted to be like that. So I decided to take part in this program. Although at first I was puzzled that homework to research 200 mathematical words, I was able to read mathematical problem sentences at next lesson. After that we learned the new way of thinking of calculation and expression transformation that is English speaking country style. I have been having fun since this season.

The problem we work on is determined, looking at it, it was a little bit difficult for us. However the biggest wall for us is not a solution but a presentation. As I thought we cannot help preparing for the presentation, I researched more details and repeated again and again in my brain. But because of that I was nervous in the presentation at Canada.

From here, I'd like to write about our travel. My image about Canada was very beautiful. There are enormous forest, a lot of fine grained snow and clean air. But actually in our trip, we went to Ottawa in other words capital of Canada. So there was no forest and for example, even if it snows, it will be removed immediately. We couldn't play with snow. These are inevitable but this is not. Surprisingly many people smoke a cigarette on the street, which rubbish is littered. I thought their conscienceness about smoking is low because drugs are legal in Canada. Therefore I felt traveling Canada is good but not good to live.

On a airplane to go to Canada, since it crossed the date change line, it seemed to be time travel and it was fun. I wasted my time on the plane by sleeping.

On the first day in the morning, we got up early and walked around our hotel. It was so cold but quickly got used to. I drank a white hot chocolate at cafe. It was too sweet for me and Canadian drinks were almost sweet. Maybe Canadians can not live without lots of sugar. Then we went sightseeing place by coach all day. We saw many sightseeing spot such as Houses of Parliament, embassy of some country including ours and historical museum and art museum. Historical museum is in front of the frozen canal in a Quebec City, and the view are so beautiful. Sunlight reflected on the canal and dazzled. Art museum was my favorite place. It was a big museum. If I had time, I would never get tired of hours. The next time I visit, I want to take a day to look around.

The second day was a presentation day. I was very nervous but Dr Angelo Mingarelli looks so

kind and his lecture about Da Vinch was really interesting. I felt a little closer when I heard Da Vinch had made lots of simple calculation mistakes. His explanation was very serious and I wanted to learn more from him. After that we went to Byward Market. When we tried to buy maple syrup, we mistakenly said four in Japanese but the clerk understood because he has a Japanese girlfriend and learning Japanese now. I was surprised but a little enviable. Various size of fir trees for Christmas are sold on the roadside and smelled amazing.

The third day was a free day, for me shopping day. I bought a lot on this day because I spent almost no money until at that time. I went to Disney store in order to find Canada Limited Edition also LEGO store, many souvenir shops and cloth shops. In the afternoon we visited Parliament Senate building which was used as a station. Because Houses of Parliament is under the internal construction.

Snow fell from night to the last day morning. The snow shined brightly with the Christmas illuminations. I'm satisfied just to see white Christmas. (I wanted to feel this atmosphere a little more if allowed.)

Incidentally I have some discovery about Canadian dishes. It means that there are very few fresh leaves. The only fresh vegetable is a lettuce of the subway sandwich. (Maybe just my luck, so this may be too much.) It was a trip I want to eat fresh vegetables like salads.

In conclusion I'm glad to take part in this program. I do think Canada trip is the best program because we can enjoy learning both math and English. And above all, I wrote various things above, but Ottawa is a beautiful town and people are all friendly. So, I'm pretty sure it was precious experience.

— S. TAKIKAWA

I am going to talk about learning through this class and a study trip in Canada.

First term, I learned that there is only one answer, but there are a lot of solution. For example when I learned mathematics in English at first, I was so surprised that how to solve an equation is difference.

I think there is only one thing how to solve an equation. But this lesson was taught for me. Because of that I am more interested in mathematics. Making a discovery as that makes me excited.

So I think it is the most important in this class that there are a lot of solution in spite of same problem.

Second term, we divide into 10 groups of 4 people and solved problems made by Dr. Mingarelli.

It was difficult for me. But we solved together, it was not difficult for us.

After term examination, we left Japan. I was uncomfortable in airplanes and tired. When we arrived at Toronto, we went through the customs. I was very afraid of that woman. We changed airlines and arrived at Ottawa. When I went out, I thought my face froze. It was very cold. The diet building with project mapping was very beautiful. I took pictures with my friend! I was excited to see revolving door when we arrived at Lord Elgin hotel. Because I saw it for the first time. I went to vending machine to buy something. There were red and blue liquid which I have never seen. I chose lemon tea. It cost \$2.50. I entered \$5. However, it came back 50 cents. I thought I was scammed! In the hotel room, there were two big beds. I was comfortable that they had large areas and bounce. I had good sleep.

Second days, I ate breakfast. It was buffet style. I think potato was the most delicious. This day we did some sightseeing in Ottawa city. I was surprised that there were many Tim Hortons. On the occasion of Carleton University, we saw rideau canal. It froze up. I wondered I wanted to skate here!

After eating lunch, we went to history museum. In the museum, An exhibition for kids was the most interesting for me. It had stamp collection. We enjoyed it.

Next we went to art museum. But I was so afraid that we had little time to see pictures. There were many interesting pictures.

After that, we went back hotel. When I went to bed, I was getting sleepy. And I dropped off. In half asleep, we went to milestone. I ordered my dinner. But it was too spicy for me to eat half. I paid five dollars as chip.

Third days, we went to Carleton university by municipal bus. I took the trouble to get 3.60 dollars. But we didn't use it on the way which we went to there. I disappointed. We gave a presentation in front of Dr. Mingarelli. I was so nervous. But he was very kind. After his lesson, we ate lunch in this university. I ate pizza. and I went to university center. I bought parker for mother. I was too surprised that there are many kinds of monster drink. On the return path I used them. We returned back and went to ByWard market. In there I ate maple candy for the first time. It was too sweet for me. But I liked it. Broadly seeing there, group of three, Kurumi, Machi, Maya and I went to Rideau center. We saw cosmetics and clothes and so on. This day's dinner was Palais Imperial. It was Chinese food. It was not that good. But desert was delicious.

Forth days, we had free time. Frist we went to Rideau center. I bought lip stick in MAC. I

gave a lot of thought to it. Next we went to cha time. This is tapioca balls and drink shop. I drunk grapefruit juice with tapioca balls. Clerk asked me what's my name. I did say "Shion". But she wrote "Shiong" on drink cup. We laughed that. After that we went to beavertail. I ate garlic cheese. It was delicious.

This day was Machi's birthday. So, Kurumi, Maya and I planted her surprise birthday party this night. We went to one dollar shop to buy balloon and a bear. Kurumi suggested that it hold skittles in its arm and surround balloon. Because She likes bears. We thought she must be glad. Then, we went back hotel. After eating dinner, we prepared birthday party for her whole she took a bath. For a while, we started party. We were glad to succeed her surprise party. We ate snack and took pictures and told a lot and play with balloon. We enjoyed party!

Last day, it was time to go japan. We left hotel and went to airport. I slept much in airplane. When I got up, CA put breakfast in front of me. For a while we reached japan! After I received carry bag, I went to eat dinner with my friend, Moe. We ate Tsukemen. It was delicious. I was very tired and sleepy. But I had a very good trip!

----- S. KOMINE

During the Canada trip, I was so impressed by very beautiful Canadian nature and scenic city. The town of Ottawa is so beautiful and I felt like I'm in a movie set. There are big green trees, big red houses, and very wide streets. All of them are very different from the landscape in Japan. I took many photos by my instant camera which can take a photo looks like very old picture. I really enjoyed taking a lot of photos in Ottawa city. The photos I took in Ottawa are not novel for the people live in Ottawa, because the subjects are very daily and usually scenes, such as a signal, supermarket, and advertisement display. Those landscapes in Ottawa may not be exciting for the Ottawa residence but it was so exciting and fashionable for me.

My friend said "I want to be a foreigner and visit Japan. Maybe we feel Japan is so scenic and fashionable, and we would take a lot of photos of Japanese city ad black hair people". I really agreed with her. Japan must be very exotic and looks like movie set for foreigners.

I plan to make the all pictures in Canada into one album. It is a big fun for me to see the pictures of my trip abroad.

I had also enjoyed very nice foods in Canada. During the trip, almost all foods are quite delicious and not extremely hot or sour. I tried having a dinner at a nice restaurant; Milestone. I had ordered my dished by myself. I and three of my friends decided to have several different

dishes and share among us. We had a lobster, stake, pasta, and salad. We had really enjoyed Canadian foods, while the volume is little too much for us.

Aside from the site seeing in Ottawa, Dr. Mingarelli's mathematics class was so interesting. At the time of the class starts, actually I was feeling tired because of long trip jet lag from Japan to Canada. In spite of hard schedule and fatigue, I really enjoyed the mathematics class and wasn't sleepy at all. His English sounds very clear for me, and it was quite easy for comprehension. The course content was very interesting which was the very famous scientists/ artists, Leonardo da Vinci. Davinci's picture is very famous. Of cause, I know his name but actually I didn't know much about his mathematic talent. It was really interesting that Dr. Mingarelli told us stories and episodes of his mistakes instead of his achievement. I had an assumption that Da Vinci never make mistake but he actually made a miscalculation!

These mistakes happen quite often but sometimes very critical, just like my mathematic exam. When I know that, I felt sense of intimacy for Leonardo da vinci.

I had experienced wonderful time at the Carleton University and I really envy the students. Carleton University has very large and so beautiful campus, so that I was feeling I was Hurry Potter. Chuo University Hachioji campus might be larger than Carleton but Carleton's campus is more attractive for me.

I know people speak English or French in Canada and may Canadians speak both relative fluently. At the by byward market in Ottawa, I found one guy at maple shop who speak good Japanese. I didn't see many Japanese at the byward market but some to the sales staffs speak Basic Japanese such as "Konnichiwa". I can imagine there were more Japanese tourer in Ottawa. Completing the Canada trip, I had experienced some essences of Canadian University. I don't know if there are any way to come back to this University as international student but I would like to visit this place again.

- R. NIKAIDO

To tell the truth, I thought I would not be able to solve these problems because they look difficult when I first look at this problem. However, by thinking the problem in steps, my group members and I became able to solve it even though the question is written in English. Firstly we began to think about the question. We used a dictionary when we found words that we didn't understand, and we managed to understand the meaning of the sentence. Actually, in this class, we have ever learned a lot of English words because our teacher has given some homework to us. The content was that we had to research and write 100 English vocabularies and its meanings in a paper. I fell it was very difficult for me to do, however, Thanks to the homework, I was able to understand some of the words that was written in our question. Therefore, I thought that I was able to read and understand our question a little easily. Eventually, we became able to understand the meaning of problem. In addition, our problem had 3 questions and they looked so difficult. However, by understanding the problem, we were able to understand it easily and we could were able to solve the question 1 by using Pythagorean's theorem. Once we came up with the way to solve, we were able to solve in steps. Actually, after we solve the question 1, we didn't need a lot of time to come up with the way to solve question 2, because the way to solve was based on the answer of question 1. Then, we were able to solve the question 2 and 3 step by step.

December 11, we have been to Haneda Airport, and have been to Canada. This was my first time to travel abroad, so I was very excited. The 1st day in Ottawa, we went to a hotel and sleep at the hotel. However, I was not able to fall asleep because I suffered from time differences. The time is about 14 hours and I was not able to fall asleep although I was not able to sleep in a airplane. The second day in Ottawa, after we had a meal, we went sightseeing in a bus and went to Carlton university. I thought that this visit was important because we would present our problem research in Carlton university in the 3rd day. We coincidentally met Angelo Mingarelli in the university. After we leave Carlton university, we went to Rideau Center, a biggest shopping center in Ottawa. In Rideau center, I wonder if I should buy something. However, I have lost the time to buy something because I didn't have confidence to communicate with people in other countries because I have never been to not only Canada, but also all countries. This was my first time to travel abroad. So, after all, I only have eaten a lunch. Then, we went to the hotel and went to milestone after we went to museum. The dinner was so delicious, and I fall asleep in the hotel. The 3rd day was very important for me because it was a day that we present our study research in front of Angelo Mingarelli. Firstly he gave his lesson in English. I had thought that his lesson was very difficult. Actually the content was difficult, but the I fell that his English was very easy to hear because it was clear. After we learned his lesson, we present our problem research in front of him. I didn't get nervous because I researched the problem in detail and practiced many time in Japan, so, my group members and I were able to accomplished smoothly, and he said that he didn't have any questions because our presentation was good. I was so glad to hear that because we tried hard to solve it and practice it. After the day, we were given the free time, so I went to shopping center or market. Then, I became able to talk and communicate to people unlike me in the day1 or day2. I thought that to communicate by using

passion is very fun.

After the training trip, I feel I could have a good experience in Canada because I accomplished the problem and I could explain it in front of teachers and students. Also, I realized the depth of mathematics and English. I think it is interesting to solve mathematics although it is written in English. Then, the most important thing that I feel in this first trip is to communicate actively even if I made a mistake in grammar. The most important thing to communicate people in other country is not to speak English correctly and perfectly, but to speak actively without ashamed. We need passion!

— M. KOMAKI

Collapsing Gas Clouds and Stability

Kazuma MITSUKAWA, Minatsu NODA,

Shoko SAKURADA, Takahisa SEGOE

Group6

Question

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P so that

$$P = \frac{3MkT}{4\pi R^{3\mu m}} - \frac{3GM^2}{20\pi R^4} \dots \text{(1)}$$

where k, G and μ are constants.

What minimum radius will the cloud start to collapse for a given mass, M, and temperature, T ?

Solution

This problem states that the mass and temperature are held constant, so the only free variable is R. We put

$$A = \frac{3MkT}{4\pi\mu m'} B = \frac{3GM^2}{20\pi} \dots \textcircled{2}$$

According to (1)

$$\mathbf{P} = \mathbf{A}R^{-3} - BR^{-4}$$

We differentiate to find the extremum and set this equal to zero.

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$$\frac{dP}{dR} = -3AR^{-4} + 4BR^{-5} = 0 \quad (\rightarrow \times R^5)$$
$$-3AR + 4B = 0$$
$$-3AR = -4B$$
$$R = \frac{4B}{3A} \quad \dots \textcircled{3}$$

We substitute (2) for (3).

$$R = \frac{4B}{3A} = \frac{4 \times \frac{3GM^2}{20\pi}}{3 \times \frac{3MkT}{4\pi\mu m}} = \frac{\frac{3GM^2}{5\pi}}{\frac{9MkT}{4\pi\mu m}} = \frac{\frac{12G\mu M^2 m}{5}}{9MkT} = \frac{12G\mu M^2 m}{45MkT} = \frac{12G\mu Mm}{45kT}$$

Answer,

$$R_C = \frac{12G\mu Mm}{45kT} \qquad \Box$$

<Impressions>

This training trip to Canada was fun all right. ...that mind wore down by going with the people who were almost the first meeting in the place not to know, and it was tired from people very much to put together.

But, in unusual environment, it spoken an unusual language, and it was fun at all pleasant chat with the staff of various shops and the people of the town. It was very good experience to be able to speak English routinely. In the beginning, own thing that I wanted to convey might not finish reaching it once, but when it was on the fourth day, I came to be able to talk about English considerably and was glad at all of own growth having been felt.

I sometimes came to like the paste of overseas people whom he/she told openheartedly in that way thought I just met very much when there was the staff who spoke Japanese.

At that time, I hesitated about everybody other telling you with local people only for the reason that a language to speak was different from oneself in and felt when it was a waste at all.

It was time when I did shopping in byward market, but it was said that it was, "You go into the shops where you went along just what of the back except the main street and was interested in more and more." from the person of other groups.

I was able to surely enter various shops, but oneself will not be suitable for a group action. I entered more shops alone and had a various talk with staff and have thought that I wanted to hear a product a lot.

After all I realize when you should go with one or a close person when you went abroad.

However, I thank two people of the member who went round various places together very much. I thought whether you were able to enjoy it because there were those two people.

I was made to do an actual feeling to came in remaining shoes in the room of the hotel abroad. I took slippers, but I thought that I wanted to feel this mysterious sense that was not felt in Japan and spent the slippers as shoes without wearing it.

Fortunately I took single occupancy, but was able to spend it comfortably so that there was not it more. I spent time of the supreme bliss to monopolizing a bed of the king size that five pillows formed a line. I felt that I did not turn to the bath though after all Japan was relaxed while thinking because it was a foreign country that a bath did not have a bathtub that hopeless.

I used it on seeing there being tea/coffee server immediately, but Canadian hard water was well correct, and it was delicious at all than tea to drink in Japan, and it was impressed by tea.

There was a lot of amenity and was thankful for there having been shoe polishing.

It is to pay a tip for bed making to have been particularly fresh. As it was an absolute thing in Japan, it was a slightly mysterious sense.

Because the meal was basically strong, and there was much quantity, I thought whether meals were not suitable me. ... which was not so delicious. It was interesting that potatoes came in anything. But thought it is a meal of gluten friendly, may disappear.

Carlton University was quite fun.

Various stationary and food and many clothes are sold in the store, and can enjoy it in the school; made it, and thought that Japan wanted to adopt such a form more.

The tacos of the school canteen were delicious. But the woman of the cash register was blunt and was slightly scared.

As a professor was very gentle and was friendly and told you like us so that even the person that English was inconvenient was plain, I enjoyed the lecture at the university plenty and listened.

Time boarding an airplane is really long, and ... which was really painful for me who had weak waist. The next person moved and was about to speak both the going and the return and was not able to sleep at all, and in-flight meal was not delicious and was really hot while I got on an airplane. I think that I want to go to Canada again if nearer.

Notre Dame sanctuary was really beautiful. Because it is before Christmas, a doll of small earthenware which imitated a moment of the birth of Jesus Christ is decorated and was pretty at all, and the lease decorated at the entrance is beautiful at all and is the moment when time when I saw this building is perfect and impressed in this trip most.

Seeing from the distance, seeing from neighborhood, I love decoration of the roof which is

particular so that a sigh is given, and is beautiful. That building which looked like both snow white and silver by how to receive natural light was really beautiful. The day when I looked at the Notre Dame sanctuary was cloudy, but innocence of the sky and the innocence of the building very much. In addition, I want to go to see it.

And the art object such as the spider in front of the National Gallery and the main building of the National Gallery, the setting sun which spread behind were beautiful surprisingly. I have taken several photographs unintentionally. Let alone the appearance of the National gallery, a display was only a beautiful thing, too. I loved sculptures, but was fascinated by a sculpture without a corner and curve useless entirely with pure white. Only with a message-related strong thing, I saw it and was fun at all so that the first was not selected as the picture either.

In this way, it is the best joy to have been able to enjoy artistry.

What it is important in this training trip to Canada that everything challenges, and do not challenge of any productivity learned that there was not it. It was quite fun.

I thought that I wanted to perform it abroad.

---- M. NODA

December 11, 2019, we got on a plane from Haneda to Toronto. Then we changed planes from Toronto to Ottawa. I have never been to Ottawa before, so I was very excited at our trips. When we arrived at Ottawa airport, I was surprised at the cold. The maximum temperature of Ottawa was minus four degrees. We stopped by the House of Parliament before we go to our hotel, Lord Elgin. Then we could see a beautiful illumination. The next morning, we went to Carleton University. It has not only vast grounds but also many buildings. In addition, There are many tunnels in Carleton University, so we could move to classroom very easily and comfortably. We could also see Dr. Mingarelli Angelo. He welcomed us with a gentle smile. His smile was very wonderful. After that, I stopped by a library of Carlton University. There were a lot of students as well as many books. I heard that they study very hard for their tests. The library has shower rooms because they stay overnight there. At noon, we moved to Rideau center to eat lunch. I ate hot dog there. It has much mustard, so I could not eat very much. After lunch I went to the shops to buy gifts for my family. I bought chocolates and maple syrups. In the afternoon, we went to historical museum of Canada. Through our visit, I could learn history of Canada. We also went to art museum. There were many beautiful works of art. I was very tired at the day. In the evening, we went to restaurant called Milestone. Then I could order my menu. I had salmon dishes which has rice and vegetables. It was very delicious. The next day, we went to Carleton University to give a presentation to Dr. Midgarelli Angelo. At first, he talked us about Leonardo da Vinchi. I could understand what he said. I was surprised that Leonardo da Vinchi made mistakes in calculation. After his lecture, we gave our presentations to him. I think I could make myself understood to him in English. But I could not answer his question because I didn't know what to say to him. Through this experience, I think I need to study more English. After the presentations, we moved to dining hill to eat lunch. Then I ate pizza which has salami and tomato ketchup. It was good. I also went to kiosk of Carleton University. I bought notebooks and pencils to my sister and cousin. In the afternoon, we went to Byward market. There were many shops which sells maple syrup, chocolates, cookies and so on. Then I bought caramel and milk chocolates to my club members. I belong to tennis club. They are very friendly. I stopped by bookstores with my group members. There were many books written in English or French. In Canada, there are two official languages, English or French. In the evening, we went to Chinese restaurant. I ate many Chinese dishes with my friends. The third day, I went to natural museum of Canada with my group members. There were a various displays such as, dinosaur fossils, insect, mineral and so on. Especially, I was very excited at dinosaur fossils like tyrannosaurus, triceratops and allosaurus. They were very big and large scale. Also, I was impressed with environmental issues. These day, there are many environmental issues in the world. I think we should try to solve them with everyone in the world. In the afternoon, we visited the Upper House of Canada. I visited foreign Upper House for the first time in my life. There were many chairs and desks. In addition, it has interpretation rooms where someone interpret English or French. In the evening, we ate steak at our hotel. It was delicious, but I could not eat very much because I was very tired. The next day, we moved to Ottawa airport to return to Japan. Through our trips, I learned not only Canadian history but also Canadian culture. In addition, I think I need to study more English and mathematics. This is because if I speak English well, I will be able to talk many foreigners when I travel in the world. And mathematics is very important for me because it is connected to science like chemistry and physics.

-T. SEGOE

This study trip was my first time to go to Canada. So, I was surprised that there were a lot of things different from Japan in Canada.

Day 1. I had never got on an airplane. I was very nervous but I could enjoy the time. Arriving in Ottawa was evening. It was very cold and very dry. I thought that Canada has so many people

but Ms. Yamaguchi, a guide of this trip, taught us that Canada's population is about one-third of Japan's. I was so surprised to hear that. I Also surprised was projection mapping of Houses of Parliament. It was very beautiful and very moving. A lot of designs of animals were shown. And projected story was wonderful. I was so moved. We had stayed at Load Elgin hotel. I was very surprised at the size and splendor of it. The hotel was very gorgeous. I like the hotel !

Day 2. We went museum and art museum. First, museum was very interesting. We could learn about a lot of foreign cultures. We also learn about the history of Canada.

Second, art museum. I didn't understand well. We sew photo exhibition called [Hanran]. But, I can not understand what was flooding. It was very difficult for me to understand the meaning.

At night, we went to restaurant [milestone]. The atmosphere of the restaurant was very good. But I didn't feel delicious. I was a little disappointed. But I want to go again and I want eat a different dish.

Day 3. We went Carlton university. Dr. Mingarelli gave us Leonardo da Vinci lesson. It was difficult for me to understand the lesson. Next, we made a presentation of our problems. It was very nervous, but I'm glad I was able to make a good presentation. This day we used local bus to go to the university. The bus was not good. We went to Byward market to buy souvenirs. There were many kind people so I can buy at ease. It was a nice place.

Day 4. It rained that day. The most surprising thing was that no one to putted up an umbrella. I really surprised that things. Our team walked wet and went to Rideau center. Rideau center was like an AEON mall in Japan. There were a lot of Interesting shop so I could enjoy the shopping. Next, we went to Houses of Parliament. The building was very gorgeous and beautiful. I love the building.

Last day of Canada trip, it was snowing. It was very beautiful and very very cold. I could enjoy the view very much.

Through this trip, I learned a lot of things. I do not like bread and pasta so I could not enjoy about the food because Canada has only wheat products such as bread. But the maple syrup that eat in Canada was very delicious. I was happy to eat very delicious maple syrup.

I could enjoy this trip and I could improve my academic ability. If I can study abroad I absolutely choose Canada and if possible I want to go to Carlton university. Even if it does not work I want to go to Ottawa again for sightseeing.

— S. SAKURADA

I will wright about our trip impression. In December 11th, we took of Haneda airport. I had not ever experienced such a long fright. I was very tired at that time. After arriving to Ottawa, we went Canada parliament. Canada parliament was lighting up. It was very beautiful and fantasy. Day one was very bored schedule. In day two, we went around in Ottawa. I will introduce three sightseeing spots in Ottawa.

First, we went to Carleton university. This university has very large area. But, this large area is said that ordinary size university in Canada. This size is incredible in Japan. In the first time, I think that I will get lost in this university by too large area. I was surprised this large area.

Second, we went to Canada history museum. There were a lot of activities in Canada museum. For example, native Canadian culture, Canada nature and Canada local specialty. I could not understand Canada history. Because they were written in English. It was very precious experience.

Third, we went to Canada art museum. There were a lot of famous pictures. I could enjoy in that museum. In day two night, we went to milestone. It is high price restaurant. I ate sirloin steak at that time. It is very delicious steak in my life.

Day two is very value day. In day three, we went to Carleton university. We had studied math in English. We listened Dr. Angelo Mingarelli class. Mr. Mingarelli English can be listened better. But, I can not understand his lesson contents. It is too hard to understand. After his lesson, problem announcement began. Our classmate looked nervous. Our announcement was very successful. Dr. Mingarelli have our announcement praise. In this night, we went to Chinese restaurant. It is not very good taste. I do not want to go this restaurant in my life. In day four, we were allowed free action in Ottawa city. Our group went to Rideau center. We were looking for present for our friend in Japan. I could look for present many time. In the afternoon, we went to Canada parliament. But, the parliament that is always used was constracting at that time. So, we went to temporary parliament. It is very large and severely security. I studied from parliament. Canadian people can speak not only English but also French. I think Canadian people is very smart. I am envious of Canadian people academic ability. In day four night, we ate beef in Lord Elgin hotel. It is very delicious. Lord Elgin hotel has excellent service and good room. But, there was regretful thing in Lord Elgin hotel. For example, every day breakfast is the same. I want to change menu every day. This trip seems long and short. Through this trip, I studied two very important things.

First, I studied the importance of English. English is used the most in the world. If I can speak English, I am very cool and smart. From these reasons, I think to speak English is very important. Second, to challenge everything is very important. These two things, I studied from Canada trip. This trip brings to me precious experience. — K. MITSUKAWA

Pythagorean theorem on Spherical surface

Chiharu UEHARA, Nanako NODA

Seiya HARADA, Hiromu FUKUDA

Group 7

Question

State and prove the classical Theorem of Pythagoras. Then state such a theorem for a SPHERICAL triangle. Can you prove it? Also, can you prove an equivalent theorem in HYPERBOLIC geometry?

Solution

(Flat)

[Method1]

First, set the length of side and vertex like figure 1. We can express two ways that solve the trapezoid.

$$(AC + ED) \times CD \div 2$$

$$= (b + a) \times (a + b) \div 2$$

$$= \frac{(a + b)^2}{2} \cdots (1)'$$

$$2 \quad 2 \quad 2 \quad 2$$
$$= ab + \frac{c^2}{2} \cdots 2'$$

(1) (and (2)) (are the same value.)

$$\frac{(a+b)^2}{2} = ab + \frac{c^2}{2}$$
$$a^2 + 2ab + b^2 = 2ab + c^2$$
$$c^2 = a^2 + b^2 \cdots 3$$

By ③ of value, we can say the "Pythagoras Theorem"

[Method2]

First, set the length of side and vertex like figure 2. Area of the triangle is "S".

$$S = \frac{1}{2}ar + \frac{1}{2}b + \frac{1}{2}cr \text{ (separate 3 areas)}$$
$$= \frac{1}{2}r(a + b + c)\cdots \text{ ()}$$
$$r = CD$$
$$= \frac{1}{2}(a + b - c)\cdots \text{ (2)}$$
$$ab = 2S$$

For ①,

$$2S = r(a + b + c)$$

For (2),

$$r(a + b + c) = \frac{1}{2}(a + b - c)(a + b + c)$$
$$= \frac{a^2}{2} + \frac{ab}{2} - \frac{ac}{2} + \frac{ab}{2} + \frac{b^2}{2} - \frac{bc}{2} + \frac{ac}{2} + \frac{bc}{2} - \frac{c^2}{2}$$

Therefore,

$$ab = \frac{a^2}{2} + \frac{ab}{2} + \frac{ab}{2} + \frac{b^2}{2} - \frac{c^2}{2}$$
$$ab = \frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{2} + ab$$
$$+ b^2 = c^2$$

As a result, this equation is correct.

 a^{2}

(Spherical case)

From figure 3, $\triangle PQR$ is a spherical triangle and $\angle PQR = 90^{\circ}$

r is radius of sphere. θ is angle between OP and xz plane and ϕ is angle between OP and xy plane. We can express P < r, θ , ϕ >

Q < r, 0, 0 > .

Converting $\leq r, \theta, \phi \geq to(x, y, z)$.

Then, set S that is perpendicular from P to OR. And we can get

 $PS = r \sin \phi$ $OS = r \cos \phi$.

So we can express $P = (r \cos\theta \cos\varphi, r \sin\theta \cos\varphi, r \sin\varphi)$

Q = (r, 0, 0).

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We can not prove this like plane, so we use thinking of vector.

 $\vec{P} = (r \cos\theta \cos\phi, \sin\theta \cos\phi, r \sin\phi)$

$$\vec{Q} = (r, 0, 0)$$

Set α the angle between P and Q.

Since $|\vec{\mathbf{P}}| = r$ and $|\vec{\mathbf{Q}}| = r$,

$$\cos \alpha = \frac{\overrightarrow{PQ}}{rr} = \cos\theta\cos\varphi \cdots (1)$$

Set PQ = c. We can get $c = r\alpha$, then we have $\alpha = \frac{c}{r}$. Similary,

we set $R < r, \theta, 0 > that PR \perp QR, PR = a, QR = b, \varphi = \frac{a}{r}, \theta = \frac{b}{r}$.

Therefore, from (\underline{l})

$$\cos\frac{c}{r} = \cos\frac{a}{r}\cos\frac{b}{r}.$$

This is theorem.

(Hyperbolic case)

Sorry, we can not understand this problem. It is too difficult for us.

 $\cosh a = \cosh b \times \cosh c$

Figures





<Impressions>

It was the first time for me to go abroad. At first, I was not very enthusiastic about this class because I wanted to another class. But I decided to take it seriously because I thought it was a good match. I was not used to learning mathematic in English and had a hard time at first, but I gradually learned to enjoy it. When our problem that we would solve was decided, we research the hint and answer, so we stopped. But it flashed when I saw the manuscript the teacher gave us that made by two year old students. I was able to read and understood, so finally solve it.

When I arrived in Canada, my breath was white and I realized that I'm in the foreign country. The guide's talk, the bus on the right and the traffic lights were all interesting for me. I was surprised by many things were written in English and French. My heart danced when I saw the loaf of the houses were sharp and frozen Rideau Canal because of snowy weather. The hotel was very clean and comfortable. I was shocked by Canadian food's volume. I couldn't eating all. We entered Carleton University and walked around and touched the snow. It was a wonderful feel like sugar. I didn't get used to Canadian cold, so I was surprised who was wearing a T-shirt. We went two museums in Canada and there made me used to using my English. When we arrived "milestone", I feel very tired. The dinner was little salty, but delicious. At night, I prepared for the next day's presentation with little not only be impatient but also fun.

Dr. Mingarelli was looks kind. His class was about Leonardo da Vinci, all in English. But I could understand most of it, even if I missed meaning. I was surprised that Leonardo da Vinci was write right to left and like mirror and he made many mistake even though he was very genius. I was interested in Leonardo da Vinci more. Our presentation was successful. I was very happy to hear he says, "It was good presentation." We thought he would ask questions, however,

he gave us some advices. He taught about Hyperbolic Geometry case. His description was polite. It still difficult for us, but I was happy. And I thought I'm glad I came here. The class was very interesting.

I used English the most when I went shopping. I wasn't used to spending Canada dollars and sometimes I was confused, but everyone in Canada were kind and some people helped me. Actually, I didn't want to go to abroad, but now I think I had good experience. I want to visit Ottawa someday. There is beautiful and kind city. I didn't eat beaver tail because I was too tired to eat sweets. So I want to have relaxing time if I go next time. It was wonderful 5 days that I would like to visit again if I have the chance.

- C. UEHARA

I went to Ottawa, Canada from December 11th to 16th. Ottawa is Canadian capital. There are mainly Houses of Parliament, Rideau Centre and Rideau canal in Ottawa.

The first day, I left Haneda, Japan. I went to Ottawa via Toronto. I was on two airplane for about 14hours, so I was bored. I was surprised that the domestic airplane is very small plane like a bus.

When we arrived at Ottawa, I felt very cold. After I got on the bus, I was surprised at driving on the right-hand side of road. I didn't get used to this point still finishing this trip. When we arrived at Houses of Parliament, we went out of the bus and took photos of it and people. My hand was painful because of too cold. Gloves weren't useful.

I felt beautiful at arriving at the hotel. And when I arrived at the room of it, I was surprised at large and beautiful room. Then I took bath. I was confused by shower because I didn't know adjustment method of water temperature. I felt "Very hot!" and "Very Cold!" alternately. After I took bath, I wanted to dry my hair. I thought using dryer, but dryer didn't move. I was a little impatient. I finally found switch of dryer and dryer could be used.

The second day, I woke up and looked out the window. Canadian noon seems to be short, so there was still dark like night.

We went to Carleton University. When I got on elevator, I thought that that elevator is large. That elevator could hold as many as 40 people. This number is the number of one class. I felt like riding an attraction. I thought why is it so large. When I passed the underground rood of university, I thought very colorful. Wall of that rood was pained variety pictures. The road of that rood was complexity, so I thought that we got lost.

We saw Rideau canal. This canal was freeze, so Canadian can skate in too cold season.

At lunch, we went to the food court. I was nervous because I payed money in Canada for the first time. I ate salad, rice and chicken. It was good, but very much. I ate it my best. I noticed that many food court in Canada have "sushi shop" during this trip, so I was delighted. When I went to the toilet in Canada at first time, I was interested in door because toilet's door is shorter than Japanese. I saw Houses of Parliament from Canadian museum of history like a castle.

At dinner, we went to Milestone. Of course Menu of Milestone was written in English, so we expect what kind of taste. It was pleasant to choose menu but it was difficult. We chose two dishes of salad, a plate of steak and a dish of pasta and these were very delicious. We calculated chip to be about 15% of all. Then we went to Rideau Centre. I noticed that people got on both sides of the escalator difference from Japan. Japanese often get on right or left side of escalator, so I thought that Japanese is impatience. I should make coin at Rideau Centre but it was more difficult than what I thought.

The third day, we went to Carleton University again. When I bought a bottle of water, I was scared because Canadian water is hard water. We studied mathematic at college. We were learned about Mr. Leonardo from Dr. Angelo Mingarelli. We presented group's problem. My group had problem that we didn't solve, so teacher told us how to solve that problem. It was difficult to understand how to solve in English for me, but I thought I had precious experience. After that we ate lunch at college. I saw machine which issues tickets to order food could be paid only credit card, so I thought that cashless progress.

When we got on the public bus, I felt be sick. Canadian road is very bumpy because people do snowplow in winter. In summer, bumpy road often be repaired. The bus made me feel sick for shaking and hot. And we arrived at Byword market. There were shops on the side of road.

We had dinner at the Chinese restaurant. We used chopsticks at Canadian Chinese restaurant.

The fourth day, we had free time. It was raining. I was surprised that raining because it only snow in winter in Canada. We went to Rideau Centre in the morning. We arrived there at 8:40 but it opened at 9:00. I thought that "we can't enter, so we wait outside". However it opened before 9:00. Shops in it opened at 9:00. After we went shopping there, we went to MANTOVATI, Byword market. This cafe had bottle of chip which people get in chip. We went to macaron shop. This shop had maple flavor macaron. I thought that maple was Canadian. After that, we saw three black squirrel, I was delighted. When I came back hotel, I found the birthday card for me. I was very surprised and happy. We went to Houses of Parliament. Security of there was like an airport. Inside there was beautiful like hotel's lobby and center of there had big

Christmas tree. Canada has two official languages such as English and French, so legislator can questions and answers using either English or French. Translator translates English from French or French from English in real time. We can watch Canadian parliament without procedure unlike Japanese parliament.

When I ate dinner at the hotel, I was surprised at cake with candles. It was good surprise situation.

The final day, it snowed from night, so snow lies on the ground and I could see white sight. It was the Canadian sight what I thought. We headed to the airport. We got on airplane for 14 hours to return Haneda airport. I was sleeping during flying. I was sad because I had a good time during trip. I felt very fast spending time during trip. I think that good trip because I spent precious time and had precious experience.

I first wanted a Polish course. But I got off by lottery. As a result, I took the course "Learn Mathematics in English". I was worried about this course because I was not good at math. The content of the lesson was often difficult to understand. It was hard to take classes in English that I couldn't understand even in Japanese. However, by learning from words, I could understand little by little. The problem raised by Dr. Mingarell were particularly difficult and could not be fully understood. The problem of hyperbolic geometry was too high for me to study in high school and was too difficult for me. My usual classes were difficult and hard, but my trip to Canada was very meaningful and fun. I have traveled abroad several times, but is the first to leave Asia. Traveling by air in 12 hours was long and tough. After the final exams, I wasn't very reluctant. I left Japan at night and arrived in Canada the same night. It was mysterious to experience the time difference for the first time. It seemed like I had time. I was surprised at the speed of immigration officers at the time of immigration about Toronto. I was able to answer the question by hearing the last word, but I don't know if it was answered correctly. I realized how slowly the ALT usually talks with simple expressions. We were stalled for nearly 15 minutes because we didn't respond well. About an hour from Toronto, we reached Ottawa. When I got out of the airport I was surprised by the cold. The air was so cold that my nose hurt when I inhaled. According to the guide, the temperature is about minus 10 degrees, and the sensed temperature drops about 3 degrees by the wind. The snow wasn't piled up as expected. The city has few street lamps and I feel lonely and less number of people than Tokyo. The Houses of Parliament were projection mapping and beautiful. But it was too cold and I had to return to the bus immediately. The hotel was clean and the beds large. It was a pity that there was no bathtub.

On the first day, I couldn't sleep very well because I slept on an airplane. On the second day, I toured the city by bus. I first went to Carlton University. The university seemed to be large and well equipped. The college tunnel connected buildings from one building to another wider than expected. The college students were all very tall and taller. I understand why Japanese cannot win physique sports. I had lunch at a food court in a shopping mall. I was relieved to be able to ask for what I wanted to do with gestures. It was hard to eat a large amount of corn and noodles because the normal size was about twice that of Japan. We went around the city government facilities by bus. The appearance of the Japanese consulate building was disappointing to consulates in other countries. The guide told me that the Rideau Canal was completely frozen but the ice was not thick enough to skate. The river was not yet completely frozen. In the afternoon, I went to the museum. To be honest, it was boring. It displayed Canadian indigenous people's lives and Canadian history, but It didn't affect my heart. The view outside the museum was exceptionally beautiful. I don't remember much at the next art museum where I was sleepy because of jet lag. Only the spider-like objects outside are impressive. For dinner, we ate seafood at a restaurant called Milestone. I was able to order what I wanted and the taste was good. The calculation of tip and state tax was difficult. After a day, I felt Ottawa was a very good city. There were not too many people like Tokyo, and the hard water that I was worried about was not much different from Japan. I couldn't be afraid that the other person could understand what they wanted to say if they could hear only the words. Although drink vending machines were rare in Canada, stereo vending machines were prominent. Perhaps there is no custom in Canada to buy electronics at electronics stores. There were also some shops that originated in Japan. There was a sushi shop, a lunch box shop, Meisou. Then there were a lot of fast food restaurants, Starbucks and Tim Hortons.

On the second day, I went to college and met Dr. Mingarell. He was a friendly person and his English was easy to hear. His story of Leonardo da Vinci was very interesting. We gave him a live problem study presentation. Dr. Mingalel explained what we didn't understand. I couldn't understand it perfectly, but I understood the rough steps and formulas. After eating lunch pizza at college, I bought souvenirs at the Byward Market. Driving the return bus was terrible, and even in Japan it was a complaint. I was hoping that the Australian market I visited earlier was very large, but it was smaller than I expected. There I bought maple syrup. Maple trees are also on the Canadian flag, and in cold Canada, sweets like maple syrup are indispensable. Dinner

Chinese food was the worst Chinese food I have ever eaten.

The third day, I had tree time and I enjoyed shopping at the Rideau Center. In the afternoon, we went to visit the Parliament Building, but it was a pity that we could only see a temporary building during construction.

This trip to Ottawa was very meaningful to me. Even if I lived here, the facilities were substantial and seemed to be fine. However, I thought this town was not suitable for sightseeing. There were no notable sights, and most were used by locals rather than tourists. If I go sightseeing, I thought Toronto or Vancouver was better. However, this time it was a training trip, so Ottawa was suitable. I thought it was an experience that could not be easily done such as giving a presentation in front of a local university professor. I could experience many things that I couldn't do in Japan. I satisfied with this Ottawa trip throughout. Six months ago, I was not motivated by the course of my second hope, but I think that this course was good.

— H. FUKUDA

I felt a lot about research and to go Canada. I want to write them here in three parts.

First, here's what I thought about working on the problem which about math with team members. My team members are configured 4 people. In September, the team talked and decided which issues each would be responsible for. At that time there was still little team communication. I was in charge of Pythagoras when it is flat. My problem was very simple but, another problem was very difficult. I could not understand and could not cooperate. I regret what I couldn't do today. I was very nervous at the presentation at the university. The presentation was finished without incident. To be honest, this research was useless for me. Because, I was just solving a given problem. And I could not try. I want to study what I was interested in. I chose science in my third years of high school. So, I would like to work hard and challenge various problems.

Second, here I write what I thought of going to Canada. I went to Canada for the first time. The time on the plane was long and very boring. Outside was very cold. The hotel was beautiful and very relaxing. But I hate early morning gatherings. The tour at the Parliament Building was a very good experience and interesting. At the Rideau Center I was able to buy what I wanted and had a very meaningful time. I had a conversation with an aunt selling souvenirs at the byword market and realized that my English ability was poor. I thought I had to study more English at this time. I am also grateful to the guides involved in this trip.

Third, here I write about my thoughts for my future. My dream is airplane pilot. Because my father is airplane pilot. When I see the pilot of Air Canada, I felt very cool. So, in order to become a pilot, I will study hard English and math.

I will continue to try various things and I will do my best again in Canada as a pilot.

— S. HARADA

Spot the difference

Machi NAKAMURA, Kurumi SENO

Group 8

Question

What is wrong with the following proof.

We know that

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$
 for all $n \ge 1 \dots 1$

Replacing n by n-1, we get

$$1+2+3+\dots+(n-1)=\frac{(n-1)n}{2}\dots *$$

Adding 1 to both sides of * gives

$$1+2+3+\dots+n = \frac{(n-1)n}{2}+1$$

Therefore,

$$\frac{n(n+1)}{2} = \frac{(n-1)n}{2} + 1$$
$$\rightarrow n = 1$$

Solution

We will explain why (1) become practical.

First we switch order of formula of the left side of (1)

$$1+2+3+\cdots + (n-1) + n$$

$$n + (n-1) + \cdots + 3 + 2 + 1$$

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We add those two formula and get

$$(n+1) + (n+1) + \cdots + (n+1) + (n+1)$$

There are *n* items so it going to be n(n+1)

We added two same formula so divide by 2

$$\frac{n(n+1)}{2}$$

Replacing n by n-1, we get

$$1+2+3+\dots+(n-2)+(n-1)=\frac{(n-1)n}{2}\dots*$$

Adding 1 to both sides of * gives

$$1+2+3+\cdots+(n-2)+n = \frac{(n-1)n}{2}+1$$

If only we add 1 to both sides,

left side of * doesn't have n-1, so it can not be $\frac{n(n+1)}{2}$

Radius of inscribed circle

Sota YAMADA

Group 8

Question

An isosceles triangle has base equal to 6cm and height equal to 4cm. Find the radius "r" of the inscribed circle.

Solution

The base of an isosceles triangle is 6cm and the height is 4cm. This consist of the property of an isosceles triangle, which can be proved from the similarity of the triangle. Therefore, the length of the hypotenuse can be obtained using a right triangle consisting of a half base, height and hypotenuse. The reason why it can be obtained is that the Pythagorean theorem hold. The Pythagorean theorem can be proved in various ways. Therefore, the hypotenuse is 5cm from Pythagorean theorem. What is calculated from this is the radius of the inscribed circle of this triangle. As a property of the inscribed circle of the triangle, the center of the inscribed circle coincidence with the intersection of the bisectors of each center of the triangle. Also, the length of tangent lines drawn from points outside the circle are equal. Therefore, if the points are placed alphabetically A, B, C, O, P, Q, R, as shown in the figure, $\triangle AOP \equiv \triangle AOR$.

Proof

Common to line AO. (1)

 $\angle A$ is divided into two equal parts by line segment AO. So, $\angle OAP = \angle OAR$ (2)

From outside the circle since the length of the drawn line segment are equal. So, AP = AR. (3) From (1(2)(3), the two pairs of points and the angle between them are equal.

This similarly holds for $\triangle BOP$ and $\triangle BOR$, $\triangle COR$ and $\triangle COQ$. Here, $\triangle ABC$ is isosceles triangle, so OQ \perp BC. Therefore, OR \perp AC and OP \perp AB, and OR = OP = OQ holds. From these conditions, the radius of the inscribed circle is obtained from the area of the triangle. If the length of the radius "r", it can be said that...

$$6 \times 4 \times \frac{1}{2} = (5+5+6) \times r \times \frac{1}{2}$$
$$24 = 16 r$$
$$r = \frac{3}{2}$$

"The proof of Leonardo's theorem"

Jo TAKAMIYA

Group 8

Question



Solution

First of all, I want to know a few formulas.



I'll to use a diagram below to prove it.

The area of a large is

$$S = (a+b)^2 \cdots (4)$$

Or
$$S = c^{2} + 4 \frac{1}{2} a \cdot b$$
$$= c^{2} + 2a \cdot b \cdots (5)$$
From (4) = (5)
$$(a+b)^{2} = c^{2} + 2ab$$
$$a^{2} + 2ab + b^{2} = c^{2} + 2ab$$
$$a^{2} + b^{2} = c^{2}$$
$$S = \frac{1}{2} a \cdot c \cdot SinB$$

I look at the area of a triangle by a triangle ratio.

SinA = height/oblique side

$$SinB = \frac{h}{c}$$

H=c · SinB
$$S = \frac{1}{2} \cdot a \cdot SinB$$

S = πr^2

First, divide the circle infinitely.



Then arrange them in parallel.

They will be infinitely equal to the π r at the bottom and r rectangle at the height. So,

$$S = \pi r \cdot r$$
$$= \pi r^2$$

Prove Problem 4 with these.



First assume AC as x, BC as \mathcal{Y} , AB as z.

By (1) $AB = x^2 + v^2 \cdots 6$ By (3) and (6), $A_3 + A_4 + A_5 = \frac{1}{2} \cdot \left(\frac{x^2 + y^2}{2} \right) \cdot \pi$ $=\frac{x^2+y^2}{8}\cdot\pi$ Assume the height of $\triangle ABC$ as h, By (2) $h = z \cdot \sin B$ $= z \cdot \frac{x}{z}$ = x $A_5 = \frac{1}{2}xy$ $A_3 + A_4 = A_3 + A_4 + A_5 - A_5$ $= \frac{x^2 + y^2}{8} \cdot \pi = \frac{xy}{2}$ $A_1 + A_3 = \frac{1}{2} \cdot \left(\frac{x}{2}\right)^2 \cdot \pi = \frac{x^2}{8} \pi$ $A_2 + A_4 = \frac{1}{2} \cdot \left(\frac{y}{2}\right)^2 \cdot \pi = \frac{y^2}{8} \pi$ $A_1 + A_2 = \frac{x^2}{8}\pi + \frac{y^2}{8}\pi - \left(\frac{x^2 + y^2}{8} \cdot \pi - \frac{xy}{2}\right)$ $=\frac{xy}{2}$

$$A_1 + A_2 = \frac{xy}{2}$$
$$A_5 = \frac{xy}{2}$$

Therefore,

So.

 $A_5 = A_1 + A_2$

<Impressions>

For the first time, I visited Ottawa in December. Before I visited there, I heard that Ottawa is very cold. However, I can't imagine how cold it is in Ottawa. It is said that highest temperature is Celsius, so I was afraid that. Actually, I arrived at airport in Ottawa and get out of the airplane, I felt so cold. I thought that I was in refrigerator. In Canada, cold wind gives us colder as well as the temperature is low. So the temperature which people feel is colder than actual temperature. On the first day of this travel, I didn't prepare for cold weather, but I had to go out to see the project mappings. I felt that my toes would froze. However, indoor is very warm, so I spent comfortable time.

In addition, our hotel was beautiful and food were great. The room we stayed was for 2 people. It is not the highest quality, but it is enough for me. I was glad that there were free wifi and beautiful bed. The hotel has also Starbucks. I regretted that I didn't have some drinks.

I satisfied with food in Canada. For the first time, I thought that Canadian food was strong and sweet taste. But actually, each food which I ate was delicious. Especially, I liked smoke salmon. It tastes fresh and strong. It was most delicious salmon that I ever ate. I ate salmon every day for breakfast. I also liked hot chocolate. There were a lot of "Tim Horton's" in Canada. I had hot chocolate at "Tim Horton's". It tastes like cocoa and very delicious. I want to drink it again. In the night, we went to restaurant every day and eat different types of food. For the first dinner, I went to "milestones". It is a gorgeous restaurant, so each menu was expensive. I ordered "Lobster & Prawn Vermicelli". It has lobster bisque. I liked this pasta very much but it was too heavy for me. In Canada, almost all food which we eat in the restaurant are too heavy. I thought that there is difference about food culture between Japan and other countries. Also, there were more and more "SUBWAYS" in Canada than in Japan. When I walk around the city of Ottawa, I saw so many "SUBWAYS". I was surprised to see that because there are a few "SUBWAYS" in Japan.

Next, I will talk about the main thing in this travel. I went to Canada to study math. I prepared for presentation before traveling, but I was very nervous. When we arrived at Carleton university, we were given a lesson by Dr. Mingarelli. He likes Leonard Da Vinci, so Dr. Mingarelli told us about him very much. I couldn't get all what he said, but he talked so interesting. I thought that he really liked Leonard Da Vinci. I was interested after his presentation. In addition, Dr. Mingarelli was very kind, so I could do presentation relaxed.

By the way, I spent my birthday in Ottawa. It was December 14. Before I went to Canada, I thought that I wanted to spend my birthday in Japan. I was born in Japan, so I wanted to stay in Japan on my birthday. However in result, I was pleased to have good birthday in Canada. I have three main reasons.

First, my friends celebrated at 0 o'clock. Just in time, they said to me, "Happy birthday!". I have never been celebrated by friends directly, so I was very happy. In addition, they celebrated at night on my birthday. My friends called for me to next to my room. When I entered the room, there were balloons on the bed and the center of there, a teddy bear was sitting with my favorite sweet! Of course I didn't know that, so I was really surprised. I thanked them very much for getting along with me and giving me birthday party. They knew that I like bears very much and what sweets I like. I again realized how important to have friends. To make things better, people who celebrated my birthday was not only friends.

Second reason why I was happy to spend my birthday in Canada is being celebrated by tour manager. At last night in Ottawa, we had dinner at Lord Elgin Hotel. We had great time with delicious meals, then suddenly tour manager said to me "Happy birthday". There were two more students who had birthday in Canada like me, so we were all celebrated. We were sung birthday song by all students. I was glad just that. However suddenly a crème brûlée with candle was carried in front of me. In addition to that, tour manager gave me chocolates as birthday presents. I didn't think that I was given a present from tour manager because I met her for the first time in this travel. I thought she didn't know even my name. I was surprised most in this travel. Crème brûlée was most delicious that I ever ate.

However, wonderful time has not end yet. Third reason to make me happy appeared after dinner. I finished dinner and went back to my room. I took off my shoes and put them into suitcase. When I squatted, suddenly I found something put on the floor. I picked it up and I was surprised. It was birthday card from Lord Elgin Hotel's stuff. I wondered why they got information about my birthday. However I was really happy, so I didn't care that. I made up my mind to keep this card forever.

Through this travel, I learned a lot of things that I can't express as words. Each time was precious for me. I will never forget this experience. I don't know if I visit Canada again but if it comes true, I will study English harder and communicate with people in Canada.

In the end, I want to thank all members who gave me special, wonderful experience. I enjoyed very much in Ottawa.

---- M. NAKAMURA

I will tell you what I learned or felt through the experiences in Canada.

First day, we arrived at Ottawa. The flight was super long and I got tired. We were taught that Ottawa is extremely cold and there was really cold. When I walked from airport entrance to a bus, it was about 10 meters, every student said "oh!! It's super cold here!!" Again and again. I wore a wide pants, a sweater, a heat tech, a down jacket and a scarf around my neck. I think it was warm enough for the temperature because it was twice as thick as I do in Japan.

Second day, we went to some places in Ottawa city. History museum had big impress on me. Why it has big impression, I think it is because the museum has many fun things. It has children's area and many totem poles. In children's area, we can learn various customs and cultures of countries all around the world. In there, theater room, we can wear costumes and go up to the small stage, small pyramid, little tatami room and so on. It was easy to learn diversity by only correcting stamps from when we are young.

Third day was our main. We went to Carlton University. We met Dr. Angelo Mingarelli to do presentation about math problems that he made. I answer the problem "spot the different". It was quite easy problem. But my teacher told me to spread more from this problem. I had no idea with it. In addition, I made a proof of each simple problem. Proving the problem that we already recognize it as it is was hard. I learned that I can study from little problem by developing it. I am so glad to know that.

We prepared the presentation since I become 2^{nd} grade. I feel I could do something more but it had been gone so I will definitely do my best in next chance.

The forth day, the greatest day was almost shopping. Actually I cannot believe how crazy I like shopping. It was new discovery. Girls were so happy with their souvenirs. I was so, too.

We stayed at beautiful hotel which was near to shopping center and Byward Market. At shopping center, I found Victoria's Secret and Victoria's Secret Pink. They are my favorite store since I was taught by Italian friend. I was looking forward to buy some fragrances there but unfortunately any smell didn't fit me. I researched about the fragrances and I had a couple of things that I want but when I go there, some of them were not in sale and others were not good. I learned that I have to try if I buy something. Recently online shopping is becoming more popular and easier to use. Therefor lots of people order cloths, foods and bags online. But if I ordered fragrances which cost 48dollars each, how loss would be made? Shopping online is very useful so I will use it in the best way.

Surprised thing that I had the most was acting in a big group. In each country I've ever been, high school students are treated as adults because it's necessary to become responsible. But in this trip most time we gathered and listened to uninteresting speech. How it works useful for me with such an birding situation? I have no idea. In addition, if 37 people gathered in a road what do you think it is going to ? Of course the people block the road. Because there are 37!! Hence acting in a group has no advantage or no good relation with education. As you know teachers have to keep students safe, so I have a suggestion. It is giving cellphone to each group to contact with teachers. Contact should be told at LINE group so that everyone can see it and verification is available in any time. Collecting all member is only before dinner at restaurant. Because of this, students should walk to the restaurant by themselves and they can get more useful information about Ottawa and they can look around while walking. I think it is much better than just walking after teachers. As Chuo University High school advocates Independent, Autonomy are very important for our development. I hope this method would be used next year.

My friend's birthday was during this trip, so we prepared a little surprise. Getting ready that was so exciting. Hopefully, the event made her happy. I think to delight someone close makes both feeling happy and warm.

Through this trip I could get many experiences both good and bad. These experiences and thoughts are my precious fortunes. I appreciate each person who let us back to home safely and make us fun with many activities. Also my family member were very glad to see me safe with souvenirs.

— K. SENO

I had a good experience participating in this Canadian research trip. One of the things I was most grateful for was that I was able to go to Canada and experience nature and culture. I went abroad for the first time in my life on this trip. Therefore, there were many things that could not be done at first. The most impressive thing is that traffic on the right is generally left-hand drive. In Japan it is generally left-hand traffic and cars are right-hand drive. I grew up in a Japanese environment for 17 years and I felt uncomfortable. Regarding the meal, I have the impression that almost no vegetables came out with bread and meat. And the amount of each one was so much different from the Japanese sense. Also, I missed rice because I never had rice. Now, here are some other impressive things. First is the difference between Canadian and Japanese personalities. The Canadians thought it was probably important to have a conversation, and they also liked to have a conversation. There are two events that felt that. The first is by bus. At that time, the road was crowded during the morning commuting rush hours. By the way, it seems that the train we just went to Ottawa has just recently been built and stops well. Therefore, many people use cars and buses for commuting to work. Back to the story, a man on the bus we were on boarded. Soon, the passenger talked to the bus driver, while driving. Surprisingly, the bus driver also responded. Prohibited in Japan. I thought, "I feel like I'm abroad" in a sight I can't usually see. The second is when I went to the market. I was at the cash register. At that time, it was crowded to some extent, but the next moment, the clerk who was walking around the store with the cashier started talking across the line where we were standing. As expected, this was shocked. You can still tell if your customers are short on time. But, as I said, the line was crowded enough to make it. I didn't know what I was saying because of my poor English ability, but maybe I was laughing and maybe not related to my job. These two events made Canadians like conversation. I thought that the attitude of actively communicating should be a Japanese practice. The next thing that impressed me was about money. Canada has no one-cent coin. Therefore the change calculation is interesting. For example, if you pay \$ 3.14 for cash of \$ 3.15, there is no change. However, if you pay \$ 3.17 for cash of \$ 3.20, you will get a change of 5cents. Because of these rules, Canadians may be a bit loose on money. When we actually got on the bus mentioned above, we took about 35 people for three people because of the inconvenience of the crew. Also, my roommates received more change than they should have. In the latter case, it may be hard to distinguish a coin from just looking at it for a moment, but it is still loose in money. This was a valuable experience that could not be experienced in Japan. Finally, I was impressed with the scenery of the Canadian city. The cityscape of Ottawa was different from the landscape I had imagined. The city of Ottawa was more French and European-like than I thought. The colors of the building were many brown and the design was caught as if you could imagine Paris. I knew the history and understood why there were so many buildings. Once Canada was a French colony and a country of its own. However, although completely independent, some states still have their remnants, so the official languages of Canada were

English and French, and were heavily influenced by France. So I found that there are many such buildings. I was looking at the scenery that I couldn't see in Japan, so I just thought it was beautiful. I think this trip to Canada was a good experience. I came out of Japan for the first time on this Canadian research trip, but I think I have expanded my horizons. If you want to interact with people from overseas, you can do it in Japan. Recently, there are many overseas people if you go to downtown such as Shinjuku. In addition, Japan is currently globalizing, and next year the Tokyo Olympics will be held. However, I felt a little different between overseas people who went to Canada this time and saw them in Japan and overseas. Therefore, it may be surprisingly important to interact with people overseas once. Also, when interacting with foreign people like that, I think that the range of conversation will expand if you know a little about that person's home country. Therefore, I think that being able to experience nature and culture abroad like this time will be a plus in my life. I realized that there are still many worlds that I do not know. I believe that it will have a positive impact on my future life and I will spend my future.

— S. YAMADA

At first, I wondered why I had to go all the way to Canada to solve this problem. You can understand how to understand English by reading a reference book. I thought it was a job to think about the solution in Japanese and translate it. That is from the point of view of presentation. The difficulty level is quite different between those who study math in the same language and have the same level of English ability in front of them and those who are native speakers of the country. I feel that the importance of the announcement in Canada was there. We practiced many times more than usual in order to have our teacher see us. The fact that we were able to do that is why the trip to Canada was meaningful to us. There are a few things that I felt during my trip to Canada. The first one is that the manuscript is not good. It wasn't as good as I thought I would use for the presentation of a manuscript carefully made over time. However, it was not possible to make the announcement smoothly because the revision was not made in time before the announcement in Canada. There are a few things that I felt during my trip to Canada. The first one is that the manuscript is not good. It wasn't as good as I thought I would use for the presentation of a manuscript carefully made over time. However, it was not possible to make the announcement smoothly because the revision was not made in time before the announcement in Canada. The second thing is that He didn't ask

me any questions. My problem is simple and it may have been worth asking. So now I think I should have chosen a more difficult problem. At first, I wasn't very enthusiastic about this class. This is because I chose it not because of my own will but because of my parents' will. That's why the teacher scolded me for my first class. I wasn't satisfied with that and I was twisting it somewhere. That feeling changed as the trip to Canada approached. I didn't want to announce what was inferior to others, and I felt strongly that I was going to Canada. As a result, I don't know if we've made a good announcement. However, I realize that I have a different motivation for this class than in April.

From here on out, I would like to talk about something a little bit different from my research. Canada was not colder than I expected. I was expecting it to be a little colder, so it was disappointing. But Ottawa seems to be getting colder in the future, so I'd like to experience it.

It was so quiet that Ottawa was really the capital of Canada. When I live in Tokyo, no matter how much I travel, buildings stand in line, cars run in large quantities, and people never stop coming and going. However, there were many streets in Canada where there were no people. However, each building in Canada was huge, and it had a scale that Tokyo could not haveI wanted to live in a luxurious house, so I thought it would be better to live in Canada.. Canada is very far and difficult to go, but I thought it was a very good country.

— J. Takamiya

Escape velocity

Takaaki ISIMOTO, Akihiro YATSUDUKA

Chihiro YAMAOKA and Moe SATO

Group9

Question

a) Show that the escape velocity, v, of a small spherical object of mass 1 from the gravitational field of a body of a spherical mass M is given by

$$\mathbf{v} = \sqrt{\frac{2GM}{R}}$$

Formulas

Law of universal gravitation

Magnitude of universal gravitation working between object of mass M and object of mass m, F is

$$\mathbf{F} = \mathbf{G} \frac{Mm}{r^2}$$

As G is Newtonian constant of gravitation $(6.673 \times 10^{-11} \text{N} \cdot m^2/kg^2)$

R is the distance between two objects.

Physical energy

When supposing that mass of object is m and velocity is v, physical energy, K is

$$\mathbf{K} = \frac{1}{2}mv^2$$

Solution

Proof

The formula for universal gravitational energy can be expressed as follows

$$U = -\frac{GMm}{R}$$

Kinetic energy K can be expressed as $\frac{1}{2}mv_0^2$

$$\mathbf{K} = \frac{1}{2}mv_0^2$$

$$U + K = -\frac{2GM}{R} + \frac{1}{2}mv_0^2 \ge 0$$

 $-2GMm + Rmv_0^2 \ge 0$ $Rmv_0^2 \ge 2GMm$

$$v_0^2 \le \frac{2GM}{R}$$

$$v_0 \le \sqrt{\frac{2GM}{R}}$$

Question

b) Given the mass M, how small should the distance R be in that the escape velocity v > c, where c is the speed of light?

Solution

Set v > c.

From a)

	$\mathbf{v} = \sqrt{\frac{2GM}{R}}$
Thus	$\sqrt{\frac{2GM}{R}} > c$
	$\frac{2GM}{R} > c^2$
	$\frac{2GM}{c^2} > R$
Therefore	$R < \frac{2GM}{c^2}$

Question

c) Conclude from b), that if the body M were a star then such a body would have to appear "black" when viewed from the outside. This is an example of a "black hall".

Solution

"Seeing". This means the light from outside researches the eyes. But this pattern from b), v > c. The right cannot thorough the body M. So our eyes cannot catch the right and we cannot see the body M. Therefore, the body M is appeared "black".

Question

d) Hoe small should the earth be in order for it to appear black to an outside observer?

Solution

Set the radius of the earth r, the mass of the earth m.

From b), the following inequality should exist in order for the earth to appear black to an outside observer;

$$r < rac{2 \mathrm{GM}}{c^2}$$

Also

$$c = 299792458 \left[m \cdot s^{-1} \right]$$

$$G \approx 6.6743 \cdot 10^{-11} [m^3 \cdot kg^3 \cdot s^{-2}]$$

Then

$$r < \frac{2 \cdot 6.6743 \cdot 10^{-11} [m^3 \cdot kg^3 \cdot s^{-2}] \cdot 5.9724 \cdot 10^{24} [kg]}{299792458 [m \cdot s^{-1}]}$$
$$= \frac{2 \cdot 6.6743 \cdot 5.9724 \cdot 10^{13}}{2997924558^2} [m]$$
$$= 0.008870399 \cdot \cdot \cdot [m]$$
$$= 8.870399 \cdot \cdot \cdot [mm]$$
$$\approx 8.8704 \ [mm]$$

Therefore, the radius of the earth should be less than approximately 8.8704 (mm) in order for it to appear black to an observer. This value is about the size of a mosquito.

< Impressions >

As for question a), the method of deriving the Escape Velocity is written in even Japanese physics textbooks and on the Internet. Also, question b), c), and d) were quite easy to solve with other members of our group, so actually it wasn't necessary to try hard to answer the questions. Furthermore, problem "P-5" we chose was the question that had been already solved by our

older students since about 2 years ago. As a matter of fact, there are the answers written by them in the school library, so we only had to cheat these answers.

In addition, there were few lectures in Mathematics in our trip to Canada. Our trip was almost 6-day trip, but the mathematical lecture by Dr. Angelo Mingarelli is only once, only about 3 hours. In other time, we went boring sightseeing that had nothing to do with mathematics. The motto of this class is "Study mathematics in English". However, there were only a few hours to "Study mathematics in English" in our trip. Moreover, it was boring that Dr. Angelo Mingarelli did not ask any questions too. I didn't see the passion for mathematics in him.

Honestly, I think this class was the waste of time for both teachers and students. I don't want to see such a boring class again.

-T. ISHIMOTO

First, I learned a lot of math-related words to solve math problems in English. This is a word related to mathematics, so it was interesting even if I was not good at English.

Little by little, I think I was able to get used to solving math problems written in English

However, I was surprised to see the problems of Professor Mingarelli of Carleton University. It is not a problem that can be solved by looking at reference books. And when I saw the second cosmic velocity problem I was in charge of, I was surprised that it was physics, not mathematics. The second cosmic speed is the speed required to escape from the earth to space. It's an interesting question, but I didn't know what to do because I had no mathematical or physical knowledge. So, I used a personal computer to investigate, and when the image was boiling, I figured out how to calculate it. It was a day of excitement and a feeling of being too difficult and painful to be able to train my thinking skills Finally, I talked with the members of the group, and I think that may understanding deepened.

I couldn't go to Canada this time. However, I have learned in the process of solving this problem. That is, learning math is more than just solving a problem. Unlike the mathematics, which only calculates by memorizing formulas, it is an interesting study that thinks about things that you do not know yet with kittle knowledge. Isn't it rare in Japan to study to train your thinking skills? In a foreign class, if I can always train my thinking like this, I will feel remorse for studying memorizing and practicing.

From now on, I would like to study not to memorize the formula but to think about it with my own head and derive the formula. — A. YATSUDUKA

This time, I went to Canada for the first time, so I was uneasy that I would confuse Canadian and I would buy the different thing.

Also, abroad was different Japan. Canadian reworded intelligibly and sought many times. I thought only presentation learning mathematics by using English was to learn but it's very important experience to interact with Canadian and expose to different culture.

Also, I thought I may not be able to understand Mr. Mingarelli's class in Carlton University but it was easy to understand because he explained figure and picture.

In addition, I learned a lot because he taught the thing I didn't learn classes in chufu. I got nervous in the presentation but he was corresponded gently. I felt it was good to do.

Also, there were many troubles in Canada. Member of 2 groups is Sayuri, Jyo, Sota and me.

Among them, Sayuri, Jyo and I were happened troubles.

First, I and Sayuri were late the time of dinner. I felt that I'm sorry.

Second, Sayuri mistook to order the draft. Then I thought I should study English a little more. But I was pleasant.

Third, Jyo lost his wallet. I learned wallet should hold dear important.

Furthermore, there were many things. The transformer was broken and the dryer was broken. The transformer is old, so I thought it could not be helped. Also, I felt I get in touch from Canada with the person being Japan easily.

We went art museum and history museum were exhibited the historic things of other than Canada.

We went to see the House of Parliament on the first day. It was cold because we went it at night. I was surprised to see it because I didn't think that its projection mapping is beautiful.

But it was hard for me to be cold.

I learned in Canada to do not like the test of Canadian food. I may have had a hard time.

But the buffet of breakfast tastes good. It was the only good thing for me.

I thought again that Japanese food is the best. The seat used of the airplane was economy class. I want to get on business class when I will be adult.

This time, it was interesting for me because ease anxiety to go with friends. So I went to go again to do new experience with friends. So I want to go to OTTAWA.

- C. YAMAOKA

I went to Ottawa, Canada. This is a first time that I went to foreign country. Our research trip

was fulfilling. But the time was too short to experience a lot in Canada. So I wanted to stay there more longer. And I have four things that I thought of spending in Canada.

First, about Carlton University. In the morning, I was very nervous for the presentation of the problem. However Dr. Mingarelli was very gentle. The beginning his class was easy to understand. Because he made the power point with a lot of figures. So I could understand what he said and I could presentation to be relax. This time was a valuable experience.

Secondly, about the people. First of all, Canadian were very very very kind. I have two episodes. The first episode is event at the drugstore. At the time of accounting, I don't know what coin I can pay then. The clerk then said, "Put a coin here." And she choose from my coin. Such behavior has been seen in various situations. So I think Canadian are kind. The second episode is an event on the bus. As a premise, Canadian buses can get drunk because the buses are very shaking. When teacher, friends and I got drunk on the bus, One man sitting in front of me stood up. I didn't know why he got up because the bus was just before the bus stop. And he said he would get off the bus soon and you can sit down here. At that time I thought he was a gentleman. Therefore I thought Canadian is kind. Dr. Mingarelli was kind too!

Thirdly, about food. Canadian food is heavy and large amount to Japanese people. For example, "salad wrap". I ate it at Carlton University. If I thought I could eat it, it was bigger than expected and had a friend eat it. The food in the restaurant is also very large amount. When we went to milestone, our group enjoyed three dishes with four people. That was just right. I want to be aware of this when I go back to Canada.

Finally, about lifeline, especially about roads. I heard "Ottawa has two seasons. That is winter and construction." from gide. But I couldn't believe that because the roads were rattling. Consequently when we got on the bus, we got dunk. There is also a reason that driving was rough. Be careful when you go to Ottawa.

Next, I want to introduce what surprised me about the difference between Japan and Canada. First, the city was very caring for people in wheelchairs. Every door has a button for wheelchair users. When they pressed that button, all the double doors were opened and fixed. Also, the slope comes out automatically on the bus. I thought that Japan should get used to it quickly. Certainly, there are many automatic doors in Japan because the climate is different between Japan and Canada. But it is certain that buses are inconvenient for wheelchair users and for us. In Japan, bus chairs are often reduced because they are sorry that wheelchairs take the time for passengers. So I think that. Seeing a different land from Japan made me aware of the lack of attention in Japan. Second, about Chinese cuisine. We ate that at dinner. It was not delicious. Because Chinese cuisine in Canada is not same Chinese cuisine in Japan. All the taste, the amount of oil, and the ingredients. But I was happy to have chopsticks after a long time. As a side note the soup and almond tofu were yummy.

Third is the amount of the Christmas tree. Christmas trees can be displayed in Japan, but that is not the same case. At the shopping mall, there was a tree here and there. There were various sizes. Also, the illuminations in the city were shining. Looking at it, I realized that Canada is focusing on Christmas.

Finally, about gingerbread cookies. I was known it but I was excited when I saw it in front of me. Gingerbread cookies were very cute with the icing of the seasonal painting. When we went it was before Christmas so there were a lot of cookies such as Santa Claus and snowman. I bought it in commemoration because I don't see it much in Japan. I ate it after returning to Japan, it was very delicious and surprising. I was icing but I didn't mind. So I like this. And I thought it is westernish.

Conclusion it's no exaggeration to say that this experience was precious.

Everything was fresh in my first experience. However That started me thinking a lot. Knowledge from experience will be a great asset. So I want to take care of the memories of this research trip. If I have the opportunity, I want to go to Canada again. Especially to Ottawa.

- M. SATO

Fibonacci sequence

Takao MARUYAMA, Hayato YOKOI, Sayumi IGARASHI Group10

Fibonacci sequence F_n is a sequence of numbers defined by the recurrence relation

$$F_{n-1} + F_n = F_{n+1}$$

Fibonacci sequence

$$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, \dots, \dots\}$$

We can create spiral from this sequence.



Fibonacci is a name of the mathematician from Italy.

It is said that the ration of the two numbers that are adjacent to each other in the Fibonacci sequence comes close to the Golden ratio as much as possible.

It is also said that Golden ratio in the most beautiful ratio.

 $\frac{1+\sqrt{5}}{2}$ This is Golden ratio.

Question

Show that if the following limit exists then $\lim_{n\to\infty} \frac{F_{n-1}}{F_n} = \varphi$, where $\varphi = 1.618 \cdots$ is called Golden Number.

Solution

Define

$$F_{n-1} + F_n = F_{n+1}$$

Divided by F_n

$$\frac{\mathbf{F}_{n-1}}{\mathbf{F}_n} + 1 = \frac{\mathbf{F}_{n+1}}{\mathbf{F}_n}$$

If

$$\lim_{n\to\infty}\frac{\mathbf{F}_{n+1}}{\mathbf{F}_n}=\varphi,$$

can be

$$\lim_{n\to\infty} \frac{\mathbf{F}_n}{\mathbf{F}_{n-1}}$$

Is one term before

If

$$\lim_{n \to \infty} \frac{\mathbf{F}_n}{\mathbf{F}_{n-1}} = \varphi = \mathbf{L}$$
$$\mathbf{L} = \frac{\mathbf{F}_n}{\mathbf{F}_{n-1}} = \frac{\mathbf{F}_{n+1}}{\mathbf{F}_n} = 1 + \frac{\mathbf{F}_{n-1}}{\mathbf{F}_n}$$

According to,

$$L = \frac{1}{L} + 1$$

We multiply L to both sides,

 $L^{2} = L + 1$ $L^{2} - L - 1 = 0$ $L = \frac{1 \pm \sqrt{5}}{2}$

Since L > 1

We get

 $L = \varphi = 1.618 \cdots$

<Impressions>

Fibonacci sequence is my topic in this lesson. When I saw the problem of Fibonacci sequence for the first time, I was very shocked. Because the problem was very difficult for me. Many mathematical symbols that I did not know were on the paper. I did not think I can solve the problem. I was trying to solve the problem but I could not find some points to solve the problem. Thank you for my teacher and my teammate. They gave me many hint so I could understand the problem and the solution. I researched much information about Fibonacci sequence to solve the problem. Fibonacci sequence and golden ratio has a deep relationship. I researched much information about golden ratio. Golden ratio is called the most beautiful ratio all the over world. Golden ratio is around us in many places. For example, sunflower, typhoon, spiral shell and so on. I learn many interested things through solve the problem. I think I cannot do any works without basic knowledge. I need to study harder. I did not go to Canada. But I learned many things and enjoyed this lesson very much. I want to study more things about Fibonacci sequence and golden ratio. I will study harder and enjoy my life in the future.

- S. IGARASHI

Golden ratio is called most beautiful ratio. Many products have golden ratio. For example, box of cigarettes, credit card and so on. It is very interesting. I learned we need to study math because you will use math in many situation. This experience is very important for me. I will to leverage this experience in life.

— Н. ҮОКО

<Pictures>



We travel wonderful country by this plane.



Ottawa is very cold.

We arrive in Toronto.





Projection mapping is beautiful.



We stay at Load Elgin Hotel.



They look like max Figure probably sleepy.

Our breakfast is a buffet. We can eat as much as we want in morning.



AA

There is rainbow road.

IC





We became chubby because of big size snack.

We arrive at Carleton University.



Children's museum is pleasant.



He call his mother "Mommy!!".



Beautiful scenery.

Scenery of outside of museum.





We arrive at an art museum.

-252-



Find a big Christmas tree in art museum.



We choose dinner by ourself.

We go to the milestone.





Nice smile with delicious dinner.



Canada's dishes is bigger than Japan's one.



They eat up every thing.



They look tired because of shopping.



We talk any plans tomorrow.



They are exciting.

the part



.....

We take a lecture at Carleton University.

They enjoy with Mr. Akiyama.



Introduction about Dr. Mingarelli.



Professor Mingarelli's lecture.

and and



Group3 give the presentation.

Dr. Mingarelli draw a diagram.



Group8 give the presentation.



Explain by using shape.



Group2 give the presentation.

Group6 give the presentation.



Group1 give the presentation about problem10.

-257-



Group1 give the presentation about problem8.



Group9 give the presentation.

Explain by using graph.





He is clever man.

-258-



Group7 give the presentation.



Listen carefully.

Dr, Mingarelli teach about new solving.



Group5 give the presentation.



Explain by using formula.





Express one's gratitude with letter.

Group4 give the presentation.



BP

Line up beautiful building.





Beautiful illumination.

There are many national flag of Canada.

Shopping in Byward Market.



Byward market at night.



Morning of third days.



We find squirrel in the city.

It's raining.



Big Ottawa monument.



We arrive at houses of parliament of Canada.



Beautiful picture of Queen Elizabeth.

E

Find a big Christmas tree in house of parliament.



Cute carpet.





Delicious steak at hotle.

Last dinner.

Successive generation of leader of Canada.



We leave Canada.



It's snowing last day.



Group photograph at history museum.

Group photograph at Ottawa airport.


Group photograph at art museum.



Group photograph with Dr. Mingarelli.



Group photograph at Ottawa airport.

Group photograph at houses of parliament of Canada.

ご旅程表

観光庁長官登録旅行業 第64号



作成日 2019年11月26日
担当者 野口 浩司
参加人数 生徒34名 教職員2名 添乗員1名
ご日程 2019年12月11日(水)~16日(月)

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中央大学附属高等学校 2019 年度教養総合(カナダ) オタワ・カールトン大学体験授業

	月日	地 名	現地時間	交通機関	行程	食事	
1	12/11	羽田空港集合	16:00	各自	集合	夕:機内	
	(水)	羽田空港発	18:50	AC002 便	エアカナダトロント直行便にて出発		
		トロント空港着	16:50		入国審査・通関後、カナダ国内線へ乗り継ぎ	昼:機内	
		トロント空港発	19:10	AC464 便	オタワヘ		
		オタワ空港着	20:11		弊社係員がお出迎えし、専用バスで市内のホテルへ	タ:〇	
		オタワ空港発	21:30		軽食配布(ハムサンドイッチ)	(軽食)	
2	12/12	オタワ		専用バス	日本語ガイドにてオタワ市内見学	朝:〇	
	(木)						
					09:00 ホテル出発	昼:×	
					※オタワ市内を車窓見学		
					09:30 カールトン大学下車(キャンパス見学)	夕:×	
					11:15 徒歩にてリドセンターにて移動		
					11:30-12:45 ランチ(各自)		
					13:15-15:30 歴史博物館		
					15:45 美術館入館		
					17:00 美術館出発		
					※バスにてホテルへ		
					18:00 ホテルロビー集合		
3	12/13	オタワ		路線バス	朝食後、路線バスでカールトン大学へ	朝:〇	
	(金)			(各自支払い)	午前中:体験授業(学校様アポイント)		
						昼:×	
					大学のカフェテリアで各自昼食後、		
					市内中心部で自由行動【予定】	夕:〇	
					18:00~ Palais Imperial (友食 山華)		
4	12/14	オタワ	終日		市内自由行動	朝:〇	
	(土)				15:30~20名 15:40~17名 国会議事堂	昼:×	
					夕食:ロードエルギン内ホテルにて	夕:〇	
5	12/15	ホテル発	08:00	車田バス		龃・○	
, s	(日)	オタワ空港差	08:30		「おべは、立山、いにくタンノ工作」。	171 . U	
	(11)	オタワ空港発	10:00	AC7603 便	トロントへ	尽・機内	
		トロント空港差	11:18	norooo (c		E. • 1/X/F J	
		トロント空港発	13:35	AC001 便	帰国の徐へ	夕:機内	
					(機中泊)	- 1000	
6	12/16	羽田空港着	16:55		通関後、解散		
	(月)				お疲れ様でした。		
l							

【利用航空会社】AC:エアカナダ 【滞在ホテル】 ロードエルギン

※日本とカナダ(オタワ)の冬時間の時差は14時間です。(日本の方が14時間進んでおります。)

カナダ研修旅行を終えて

大野江里子

秋山先生に同行してカナダ研修旅行に参加することが決まり,何度か授業の様子を見る機 会を頂きました。生徒達が取り組んでいる問題は高校数学を超えた難易度の高いものが多く, すべて英語で書かれている。そのレベルの高さに驚きを感じ,さらに必死に取り組んでいる 生徒たちの姿,彼らの学習意欲には目を見張るものがありました。英語は,使うためにある。 英語はコミュニケーションツールである。ということを目の当たりにした一瞬でした。

私がかつて受けた英語の授業はリスニングもなく英会話の授業を受けた記憶もありませ ん。英語とはひたすら紙に向かって学習する教科で,試験で点数を取るために勉強をしてい ました。母国語を異にする人々とコミュニケーションを取る機会も,それに対する欲求も少 なかったような気がします。それに比べ,今の中高生は,リスニングや会話を中心とした英 語の授業を多く受け,ニュースでは世界中で活躍している日本人が英語でのインタビューに 受け答えしている様子が毎日放送されています。英語を使えるようになりたい,使う機会が 欲しいと感じている生徒は多いでしょう。しかし,まだまだ日常生活において英語を使うタ イミングは積極的に求めないとありません。ですから,今回のように,英語を実際に使うこ とができる研修旅行がどれだけ貴重な体験だったかを改めて感じて欲しいと思います。

英語を学習する目的は、当然文法を理解する、点数を取ることだけではありません。真の 目的は、母国語を異にする世界中の人々とコミュニケーションを取ることで異文化に触れ、 これからのグローバル社会で生き抜くために、各人の活躍の場を日本国内にとどめず、それ ぞれの可能性を広げることです。この研修旅行をきっかけに、すでに留学に興味を持った生 徒もいるようです。また、チャレンジすることへの恐怖心や心理的な障壁も小さくすること ができた生徒も多いと感じています。この研修旅行は日常の学校生活で学習したことを実践 する場として、価値のあるかけがえのない体験となったと確信しています。

<Impression>

今回が5回目のカナダ研究旅行になる。第3回までは希望生徒のみの参加であったが、4 回目からはSSH(Super Science High School)と「教養総合 I」の正規授業として高校2年生 全員がいずれかの教養総合授業を選択しなければならないという形式に大きく変わった。以 前の希望参加生徒だけの研究旅行は放課後に集まり、英語で数学を学習していた。それなり に情熱がある生徒のみが集まり授業にも緊張感があった。もちろん参加生徒数は今の40名 近くではなく、その半数程度であった。4回目以降は希望する生徒と抽選ではずれ仕方なく この授業に参加した生徒もいたために、生徒の授業への取組情熱に大きな差が出ていた。さ らに理数系の教科は能力の差が激しく、大学3~4年生くらいの知識を持っている生徒がい れば、一方では中学生レベルの知識に加え、ゼロに限りなく近い情熱も持っている生徒がい た。参加人数40という数字は授業として40人をある一定レベルの目標に向けて指導すると いうハードルである。このような形態では「地域研究」を主とする教養総合授業では支障が 起こるはずもなく、順調に授業展開ができると考える。一方数学の知識をある一定レベル以 上にさらにはそれを英語で使えるようにするには2単位の授業では到底時間が足りない。

この授業には2つの大きな目標がある。

1) 数学を英語で表現できる

2) オタワ(カナダの首都)という地域を研究する

目標1)に関しては生徒間にかなり大きなばらつきがあり,なかなか生徒全員が同じレベルになることは厳しいことを毎回感じている。目標2)に関してはまずオタワの地理的場所を知ること。日本の首都東京との違い,首都とはどのような場所が望ましいのか,政治的街と商業的街の違いなども目標に,最終は自分でカナダ,オタワを体感することである。学校という安心して失敗できる場所でたくさんの失敗をしてほしかった。

今回の生徒達には感想文に関して 1000words 以上の条件を課した。多くの生徒たちは数学の問題を英語で考え解くことや、感想文の条件など多くの困難な task に対し、果敢に挑み良い仕事をしたと考える。

数ある教養総合Iの授業で「英語で数学」を選択した生徒たちが一番勉強したと私は自負 しており、この生徒たちを誇りに感じる。しかし、上記生徒たちに係るマイナスな原因・結 果全は秋山の能力不足に起因することを忘れないで記しておく。Act Alive.