# Study trip in OTTAWA, the Capital City of Canada

-Math lecture in English at Carleton University-

Kazuo AKIYAMA Eriko OHNO

## <Summary>

"Why do we have to study mathematics in English?" That is a common question for Japanese high school students. That was the main idea behind the study trip to Canada. Even though our students are Japanese, in their future, they may be required to study some subjects in English. That's one of the reasons why I wanted to plan this experimental trip to Canada. The other reason was that I wanted my students to experience a class at a Canadian university. I wanted my students to feel a western atmosphere, "Feel, feel, feel it." I had a good supervisor, and thanks to his support we were able to turn this plan into a reality. He gave my students ten problems, which they worked hard on to find the solutions. They thought in English, made their answers in English, and also had to present them in English. I believe it became an invaluable experience for them. We also became interested in Ottawa City, the capital, and learned about some of the differences between life in Ottawa and Tokyo. We looked around the areas of Maple Sugar Bush, and Montreal in the French-speaking province. This trip provided our students with a unique chance to experience a true sense of globalization in action.

## <Plan>

In March 2016, I wrote the letter to Dr. Mingarelli

10 March 2016

## Hi-Angelo,

How have you been, Dr. Mingarelli. I myself reached the age of sixty last month.

I would like to inquire whether you will be there in Canada around this time next March. Would it be possible to give lectures to about thirty students as you did before? I am planning to visit Carleton University again with my second grade students at the end of next March, when they will hopefully attend your lectures and explore the campus. The contents of the lectures could be the same as before, or the latest topic on mathematics.

Key words and phrases. Study trip, OTTAWA, mathematics, presentation, high School.

I am looking forward to hearing from your reply. Best Regards, Kazuo

Actually, this is 3<sup>rd</sup> time for Canadian study trip. Description of Canada study tour by the students (Mr.AWAZAWA & Miss MIYATA) has been carried out at the end of the first semester.

< 23 July > From: Dr. Mingarelli

Hi Kazuo.

It's good to hear from you again.

You are right that I will be on leave then but I should be here last week in March. I think it's okay.

Best wishes to all Angelo

< 31 Aug. > To: **Dr.Mingarelli** *Hi Angelo,* 

About assignment

Age:16-17 years old (maybe grade 12 in Canada?)

They learned

Polynomial, Quadratic equation, Higher order equation, Trigonometric function, Probability, Vector (not linear space)in two dimension and three dimension, etc.

They will learn from September,

*Exponential and logarithmic function, differential and integral function, sequence and series, etc.* 

Assignment: we have 6 months for treating your assignment. All kind of field of Mathematics. At least ten topics we need. \*problem (solvable or unsolvable) \*some short research \*How do we work with your assignments? Depend on each assignment, For tough problem or assignment, I assign students who are interesting in mathematics too much. For light assignment, I assign students who are interesting in mathematics not too much.

< 31 Aug. >From: **Dr.Mingarelli** Ok Kazuo. I'll work on this right away. Angelo

# About Carleton University and Department of Mathematics



Carleton University's roots as a non-denominational college supported in part by charitable donations from the Ottawa community make it unique among Ontario universities. Founded in 1942, Carleton was created in response to the need to help provide the young people in Ottawa, many of whom had taken on jobs to cope with the pressures of the Depression, with an opportunity to continue their formal education.

From its humble beginnings on Ottawa's First Avenue, Carleton has grown into a dynamic research and teaching institution with a tradition of anticipating and leading change. Today, the university sits on more than 100 acres, on a site between the Rideau River and the Rideau Canal, just a short distance from downtown Ottawa.

The university provides an excellent education and experience to its more than 24,000 full- and part-time students at the undergraduate and graduate levels. Its more than 875 academic staff are recognized internationally for their scholarship and cutting-edge research in more than 50 disciplines. Carleton's reputation is built on its strengths in the fields of journalism, public affairs, international affairs, architecture and high technology. Its students benefit from the interdisciplinary, active,

hands-on approach to teaching and research practiced by its faculty members and from the numerous partnerships the university has with the federal government, other universities and private sector partners.

<School of Mathematics and Statistics>

In today's world, the modern citizen is routinely confronted by a maze of numbers and data in various forms. Mathematics and Statistics are also at the heart of many of today's advancements in science and technology and are contributing to progress in other fields such as industrial and architectural design, economics, biology, linguistics, and psychology.

Since 1942, the students enrolled in the School of Mathematics and Statistics at Carleton have enjoyed the unique advantage of studying in the nation's capital, providing easy access to internship opportunities in government departments and private industry.

## **Our Programs**

- Bachelor of Mathematics
- Master of Science: Mathematics and Statistics
- Ph.D.: Mathematics and Statistics

## **Our Fields**

- Applied Mathematics
- Bioinformatics (M.Sc. only)
- Biostatistics (M.Sc. only)
- Probability and Statistics
- Pure Mathematics

## About Dr. Angelo Bernardo MINGARELLI

<u>RANK</u>: Full Professor with tenure <u>DEGREES</u>: Ph.D. University of Toronto, 1979 M.Sc. University of Toronto, 1975 B.Sc. (summa cum laude), Loyola College, Montreal, 1974



# CAREER HISTORY

1990-Full Professor with tenure, Carleton University, Ottawa, Ontario

1989-1990 Full Professor with tenure, University of Ottawa, Ottawa, Canada

1985–1989 Associate Professor with tenure and NSERC University Research Fellow, University of Ottawa, Ottawa, Canada

1981–1985 Assistant Professor and NSERC University Research Fellow, University of Ottawa, Ottawa, Canada

## HONORS

\*Faculty of Science Award for Excellence in Teaching, 1996 (Awarded, Fall, 1997)

\*Faculty of Science Award for Excellence in Teaching, 1992 (Awarded, Fall, 1993)

\*Teacher of the Year nominee for 1987, University of Ottawa

<u>GRADUATE STUDENT SUPERVISION AND CURRENT EMPLOYERS</u> (Only the part related to Akiyama)

\*Akiyama K., M.Sc., University of Ottawa, 1983; (Teacher in Chuo University High School, Tokyo, Japan)

# BOOKS AND MONOGRAPHS

"The ABC's of Calculus": Module on Inverse Functions The Nolan Company, Ottawa, 1994, 78 p. "Non-oscillation domains of differential equations with two parameters"

jointly with S. G. Halvorsen LECTURE NOTES IN MATHEMATICS 1338, Springer-Verlag, Heidelberg, 1988, xi, 109 p. (Only the part related to Akiyama)

MATHEMATICAL PAPERS IN REFEREED JOURNALS (Only the part related to Akiyama) Oscillation of linear second order differential systems (with M.K. Kwong, H.G. Kaper, K. Akiyama) Proc. Amer. Math. Soc., 91 (1984), 85–91

# <Student comment>

Angelo Mingarelli was one of the most amazing professors I have ever had. He is excellent at explaining topics and answering questions. He creates unique methods of solving problems and memorizing concepts. I really appreciated his incredible stories which made it worth attending lectures, though they were essentially the textbook read to us.

# <10 problems, answer, presentation and students' impression>

Hi Kazuo:

Here is the assignment. With my best wishes, Sincerely,

Angelo

# 10 Questions for in class presentation, March 2017

- State and prove the classical Theorem of Pythagoras. Then state such a theorem for a SPHERICAL triangle. Can you prove it? Also, can you prove an equivalent theorem in HYPERBOLIC geometry?
- 2. Define a straight-edge and compass construction in plane geometry. Give Archimedes' construction for the trisection of an angle and show why it is not an acceptable solution to this 2500 year old problem. What type of triangles can be trisected?
- Collapsing Gas Clouds Stability http://spacemath.gsfc.nasa.gov/Calculus/5Page37.pdf
- Space Shuttle Launch Trajectory I http://spacemath.gsfc.nasa.gov/Calculus/5Page40.pdf

# 5.

a) Show that the *escape velocity*, *v*, of a small spherical object of mass 1 from the gravitational field of a body of a spherical mass M is given by

$$v = \sqrt{\frac{\mathrm{GM}}{\mathrm{R}}},$$

where R is the distance between the centers of the two bodies.

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- b) Given the mass M, how small should the distance R be in order that the *escape velocity* v > c, where c is the speed of light?
- c) Conclude from b) that if the body M were a star then such a body would have to appear "black" when viewed from the outside. This is an example of a "black hole".
- d) How small should the earth be in order for it to appear "black" to an outside observer?
- 6. The Fibonacci sequence  $F_n$  is a sequence of numbers defined by the recurrence relation,  $F_{n+1} = F_n + F_{n-1}$  where  $F_0 = 0$ ,  $F_1 = 1$  and the first few terms are given by 0; 1; 1; 2; 3; 5; 8; 13; 21; · · ·
- a) Show that if the following limit exists, then

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\varphi\,,$$

where  $\varphi = 1.618 \cdots$  is the so-called *Golden Number*.

b) If  $\alpha$  is any positive integer, show that Show that if the following limit exists, then

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\varphi^{\alpha}.$$

- 7. The Close Encounter to the Sun of Barnard's Star http://spacemath.gsfc.nasa.gov/Calculus/9Page18.pdf
- Rotation Velocity of a Galaxy http://spacemath.gsfc.nasa.gov/Calculus/6Page106.pdf
- 9. Why are hot things red?

http://spacemath.gsfc.nasa.gov/Calculus/5Page36.pdf

10. Define an arithmetic progression. State the Green-Tao Theorem for primes in arithmetic progression. For n = 3; 4; 5; 6; 7 find an arithmetic progression containing n primes. What is the largest value of n known sofar for which there are n primes in an arithmetic progression?

	Student Name	Problem#	
常世田 将史	常世田 将史 Masashi TOKOYODA		
粟沢 和也	Kazuya AWAZAWA	1	
久門 美羽	Miu KUMON	1	
吉田 瑠璃	Ruri YOSHIDA	1	
前田 依莉子	Eriko MAEDA	1	
金子 葉月	Hazuki KANEKO	2	
綱島 杏奈	Anna TSUNASHIMA	2	
宮田 新菜	Niina MIYATA	2	
松本 真子	Mako MATSUMOTO	3	
山本 莉衣奈	Riina YAMAMOTO	3	
深津 佳奈	Kana FUKATSU	3	
佐竹 大斗	Daito SATAKE	4	
野崎 拓人	Takuto NOZAKI	4	
大西 佑磨	Yuma OHNISHI	4	
平 剛志	Takeshi TAIRA	4	
久門 美羽	Miu KUMON	5	
吉田 瑠璃	Ruri YOSHIDA	5	
前田 依莉子	Eriko MAEDA	5	
堀田 渓人	Keito HOTTA	6	
柳 そら	Sora YANAGI	6	
都倉 彩乃	Ayano TOKURA	6	
高波 明日香	Asuka TAKANAMI	6	
川本 修平	Shuhei KAWAMOTO	7	
武田 直之	Naoyuki TAKEDA	7	
三崎 優太	Yuta MISAKI	7	
中島 笙	Sho NAKASHIMA	7	
高田 将希	Masaki TAKADA	8	
矢崎 航平	Kohei YAZAKI	8	
中田 裕徳	Hironori NAKADA	8	
山中 侑人	Yuto YAMANAKA	8	
上村 優大	Yudai UEMURA	9	
寺田 大記	Daiki TERADA	9	
望月 駿介	Shunsuke MOCHIZUKI	9	
常世田 将史	Masashi TOKOYODA	10	
粟沢 和也	Kazuya AWAZAWA	10	

# <Problem#1: Theory of Pythagoras>

# Kazuya AWAZAWA , Masashi TOKOYODA , Miu KUMON , Ruri YOSHIDA , Eriko MAEDA

## Question.

State and prove the classical Theorem of Pythagoras. Then state such a theorem for a SPHERICAL triangle. Can you prove it? Also, can you prove an equivalent in HYPERBOLIC geometry?

Solution

<Flat>

We're going to introduce one proof we feel easy to understand.

Set the length of sides like Figure 1.

And, with four of this triangle, we can draw Figure 2.

Then, we'll express the area of the smaller square, S, in two ways.

First, the length of the side is c. So we can express

$$S = c^2 \cdots 1$$

Second, S is the area of the big square excluding the area of the hatched area.

The length of the big square is a + b

So we can express

$$S = (a + b)^2 - 2ab = a^2 + b^2 \cdots 2$$

From 1 and 2,

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a^2 + b^2 = c^2 \qquad \qquad \text{Q.E.D.}
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<Spherical case>

In this presentation, when we set a point P and set  $\rho$  the distance between O and P, we express

where  $\theta$  is angle between OP and x axis and where  $\phi$  is angle between OP and xy plane.

We can't prove this like plane. So we use thinking of vector.

Set P< $\rho$ ,  $\theta$ ,  $\phi$ > and Q< $\rho$ , 0, 0>. (They are on surface of a sphere whose radius is  $\rho$ . See Figure 3.) Converting < $\rho$ ,  $\theta$ ,  $\phi$ > to (x, y, z),  $\mathbf{P} = (\rho \cos \theta \cos \varphi, \rho \sin \theta \cos \varphi, \rho \sin \varphi)$ 

So Position Vector for P is

$$\vec{P} = (\rho \cos \theta \cos \varphi, \rho \sin \theta \cos \varphi, \rho \sin \varphi)$$

Similarly, Position Vector for Q is

 $\vec{Q} = (\rho \cos \theta \cos \varphi, 0 \times \sin \theta \cos \varphi, 0 \times \sin \varphi) = (\rho \cos \theta \cos \varphi, 0, 0)$ 

Set  $\alpha$  the angle between P and Q.

Since  $|\vec{P}| = 1$  and  $|\vec{Q}| = 1$ ,

$$\cos \alpha = \frac{\vec{p} \cdot \vec{q}}{\rho \cdot \rho} = \cos \theta \cos \varphi \quad \cdots \textcircled{1}$$

Set PQ=c, c is given by

$$c = \rho \alpha$$
$$\therefore \alpha = \frac{c}{\rho}$$

Similarly, if we set R< $\rho$ , $\theta$ ,0>, (PR $\perp$ QR, PR = a, QR = b. See Figure 4.)

$$\theta = \frac{a}{\rho}$$
$$\varphi = \frac{b}{\rho}$$

Therefore, from ①,

 $\cos\frac{c}{\rho} = \cos\frac{a}{\rho} \cdot \cos\frac{b}{\rho}$  Q.E.D.

<Hyperbolic case>

Sorry, but Hyperbolic plain was too difficult for us to understand when we tried it. So we couldn't solve this problem.

Mr. Mingarelli's question.

(In the spherical case,) The "PR" is an arch? (Not a straight line?)

-Our answer: Yes.

<Figures>



Figure 1



Figure 2



Figure 3

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Figure 4

## <Impressions>

Third day, I was tense. Because I am not good at English and math. So I was worried to answer the question from the math teacher. But it was considered badly. He was very kind teacher. I was very happy when he praised my team.

-M. KUMON

I was surprised that language was so close to us that we faced more problems when our native language wasn't well-informed. Especially, proofing was so difficult because we hardly do it habitually. So I'm relieved that we've done it.

—E. MAEDA

Third day, one more person's and my physical condition got worse in the morning. When everyone went to the Carlton University, we go to a hospital in Ottawa city with a person of JTB. We asked security guard of a building about the location of the hospital. We waited about two hours, and we got a consultation. First one more person got a consultation. It was good that I was able to understand what I was asked before I got a consultation because I wasn't good for listening English. My symptoms were caused by fatigue. I think cause of fatigue was first 13 hours flight.

For that reason I didn't join presentations.

I worked hard to solve a math problem before we went to Canada. I am not very good at mathematics. A problem that we solved with other group is very difficult. I could hardly understand. But I was enjoyable to solve with wise persons. It was a good experience that solving math problems in English.

-R. YOSHIDA

# <Problem#2: Trisection>

## Hazuki KANEKO, Anna TSUNASHIMA, Niina MIYATA

## Question

Define a straight-edge and compass construction in plane geometry. Give Archimedes' construction for the trisection of an angle and show why it is not an acceptable solution to this 2500 year old problem. What type of triangle can be trisected?

#### **Solution**

It is easy to bisect arbitrary angle with a straight-edge and a compass, to trisect is impossible to trisect with a straight-edge and a compass alone.

We can draw when using special tools.

## Using "origami"

- 1. Fold origami suitably, make a arbitrary angle. (Figure 1)
- 2. Fold twice at the same width, and make creases. (Figure 2)
- 3. Fold so that A is on A' which is on PC, C is on C' which is on BE, and make creases. (Figure 3)
- 4. About midpoint B' of A'C', fold as A'B' and C'B' overlap, the crease passes through the C. (Figure 4)
- 5. In 4, CB' and CC' give trisect of angle.
  - (3 and 4 is the point of trisect of angle.) (Figure 5)

Then, is it really impossible to draw trisected angle with compass and straight-edge? We can draw trisected angle by using trisection of angle made by Archimedes.

1. Draw the circle with radius r centered on C when given arbitrary angle  $\theta$ .

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\theta = \angle POC (Figure 6)
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- 2. Draw a line that AB=r passing through C. (Figure 7)
- 3. It's the triangle OBC is a isosceles triangle so the angle OBC=OCB, and it's the triangle BAO is a isosceles triangle so the angle BAO=BOA (Figure 8)
- 4. Assuming the angle BAO=BOA= $\alpha$ , it is OBC=BAO+AOB, so OBC=OCB= $2\alpha$  (Figure 9)
- 5. It is the angle POC=OAB+OCB so  $\theta$ =3 $\alpha$ . That is it  $\alpha$ = $\theta$ /3.  $\alpha$  is trisected angle of  $\theta$ . (Figure 10)

So, it is possible to draw an angle that trisects an angle with compass and straight-edge.

However it is impossible to trisect arbitrary angle, it is possible to draw trisected angle with compass and straight-edge. And method of Archimedes.

Comment from Dr. Mingarelli

Straight edge is not ruler.

## Our answer

We didn't know understand difference between the straight edge and the ruler.

In Japanese, straight edge and ruler is same word " 定 規". Our mistake is didn't examine the meaning of the word well.

## Solution

It is easy to bisect arbitrary angle with a straight-edge and a compass, to trisect is impossible to trisect with a straight-edge and a compass alone.

We can draw when using special tools.

Using "origami"

1. Fold origami suitably, make a arbitrary angle.



2. Fold twice at the same width, and make creases.



3. Fold so that A is on A' which is on PC, C is on C' which is on BE, and make creases.



Figure 3

4. About midpoint B of A'C', fold as A'B' and C'B' overlap, the crease passes through the C.



5. In 4, CB' and CC' give trisect of angle. (3 and 4 is the point of trisect of angle.)



Then, is it really impossible to draw trisected angle with compass and straight-edge? We can draw trisected angle by using trisection of angle made by Archimedes.

1. Draw the circle with radius r centered on C when given arbitrary angle  $\theta$ .



Figure 6

2. Draw a line that AB = r passing through C.



3. It's the triangle OBC is a isosceles triangle so the angle OBC=OCB, and it's the triangle BAO is a isosceles triangle so the angle BAO=BOA



4. Assuming the angle BAO=BOA= $\alpha$ , it is OBC=BAO+AOB, so OBC=OCB= $2\alpha$ 



5. It is the angle POC=OAB+OCB so  $\theta = 3\alpha$ . That is it  $\alpha = \theta/3$ .  $\alpha$  is trisected angle of  $\theta$ .



So, it is possible to draw an angle that trisects an angle with compass and straight-edge.

However it is impossible to trisect arbitrary angle, it is possible to draw trisected angle with compass and straight-edge. And method of Archimedes.

## <Impressions>

I was acutely aware of my poor English ability by Canada study tour. I understood only simple English after five years since I started studying English. I was sad because I sometimes didn't understand the conversation at the restraint. From such experiences, I think I don't have ability to listen to English and speak English. Thanks to this study tour, I came to think that I want to learn English at university. Next time I go to a foreign country I want to enjoy communication in English.

-H. KANEKO

I think the Canada study tour this time is very good as I could visited places and did experience not going on sightseeing. By touching a completely different culture from Japan, I realized again the wonderfulness of each country. However, I was regretful that we couldn't walk the city much in Montreal sightseeing.

-A. TSUNASHIMA

This study tour is very good experience in my high school life. I don't like study math and English. But, I could enjoy this tour. This tour is busier than other tours. I always thought "I don't want to prepare tour." However, after I think this experience is very good thing in my life. Thanks to this tour, I could learn many English. This experience have good impact. I spend beautiful time in Canada. I never forget this tour.

-N. MIYATA

# <Problem#3 : Collapsing Gas Clouds—Stability>

# Kana FUKATSU, Mako MATSUMOTO, Riina YAMAMOTO

Question

A gas sphere with a radius, R, a mass, M, and a temperature, T, is subject to an external pressure, P

 $P = \frac{3MkT}{4\pi R^3 \mu m} - \frac{3GM^2}{20\pi R^4} \dots (1)$ 

where k, G and  $\mu$  are constants.

**Solution** 

We assume that an expression ① is correct.

We put

A = 
$$\frac{3MkT}{4\pi\mu m}$$
, B =  $\frac{3GM^2}{20\pi}$  ... (2)

According to 1

 $P = AR^{-3} - BR^{-4}$ 

We differentiate to find minimum.

$$\frac{dP}{dR} = -3AR^{-4} + 4BR^{-5} = 0$$
$$(\rightarrow \times R^{5})$$
$$-3AR + 4B = 0$$
$$-3AR = -4B$$
$$R = \frac{4B}{3A} \dots (3)$$

We substitute 2 for 3.

$$R = \frac{4B}{3A} = \frac{4 \times \frac{3GM^2}{20\pi}}{3 \times \frac{3MkT}{3\pi \mu m}} = \frac{\frac{3GM^2}{5\pi}}{\frac{9MkT}{4\pi \mu m}} = \frac{\frac{12G\mu M^2 m}{5}}{9MkT} = \frac{12G\mu M^2 m}{45MkT}$$

$$= \frac{12G\mu Mm}{45kT}$$

Answer,  $\frac{12G\mu Mm}{45kT}$ 

## <Impressions>

I got many things through this travel. I had trouble to use dollar and cent. It was a first experience. And though I am not good at English, I could feel that English was fun when people understood what I was saying. I would like to study English to communicate with local person more. The scenery of Canada was very beautiful. And people of Canada were kind. So I came to like Canada. I recommend everyone to go to Canada.

-K. FUKATSU

Prepare for presentation was hard and presentation in English in front of native teacher was so nervous. However, finally, it became a good experience, it was excited. I couldn't speak enough with tension, so I want to be able to talk in English with person more. Finally, it was the first trip to Canada, I discovered various difference, they were so interesting. I came to want to know more. I think this trip was a good choice.

-M. MATSUMOTO

This travel will be in my memories forever. Especially, the views of Montreal were the most wonderful in this travel. Montreal was like Europe.It was hard for me to eat a large amount of meal. But I was happy to eat duck meat for the first time. Canadian culture is very different from Japanese culture. So I learned many things from there. I thought that English is very important and I must study English very hard.

-R. YAMAMOTO

# <Pronlem#4 : Trajectory>

## Daito SATAKE, Takuto NOZAKI, Yuma ONISHI, Takashi TAIRA

Question1:

Q. Use the parametric equations for h(T) and R(T) to determine the equation for the Speed S, of the Shuttle along its trajectory where  $dS/dt = {(dh/dt)^2 + (dR/dt)^2}^{1/2}$ .

## Solution

Gives us the equation  ${(dh/dt)^2+(dR/dt)^2}^{1/2}$ , so we differentiate the equations for h(T) and R(T) in this way.

$$dh/dt = -0.142T^{2}+36.6T-345$$
$$dR/dt = 135.7e^{0.029T}$$

Then

$$dS/dt = \{(-0.142T^2 + 36.6T - 345)^2 + (135.7e^{0.029T})^2\}^{1/2}$$

Question2:

Q. Determine the formula for the magnitude of the acceleration of the Shuttle using the second time derivatives of the parametric equations.

## **Solution**

Gives us the equation h(T) and R(T), we differentiate these equation twice.

• h(T) = 2008-0,047T<sup>3</sup>+18,3T<sup>2</sup>-345T•R(T) = 4680e<sup>0.029T</sup>  

$$\downarrow$$
 Differentiate twice  
• d<sup>2</sup>h/dt<sup>2</sup>=36.6-0.284T • d<sup>2</sup>R/dt<sup>2</sup>=3.93e<sup>0.029T</sup>  
,a<sub>R</sub>=d<sup>2</sup>R/dt<sup>2</sup>  
theorem is |a| = (a<sub>b</sub><sup>2</sup>+a<sub>R</sub><sup>2</sup>)<sup>1/2</sup>,

Then,  $ah = d^2h/dt^2$ ,  $a_R = d^2R/dt^2$ 

The Pythagorean theorem is  $|a| = (a_h^2 + a_R^2)^{1/2}$ , so, we have  $|a| = (a_h^2 + a_R^2)^{1/2} = (0.08T^2 - 20.8T + 1339 + 15.4e^{0.058T})^{1/2}$ .

## Question3:

A) Q. From your answer to problem2, find the time at which the acceleration is an extremum, and specifically, a maximum along the modeled trajectory. **Solution** 

$$|a| = (0.08T^{2}-20.8T+1339+15.4e^{0.058T})^{1/2}$$
  
When d|a|/dT=0,  
$$0 = 1/2(0.08T^{2}-20.8T+1339+15.4e^{0.058T})^{-1/2}(0.16T-20.8+0.89e^{0.058T})$$
$$\rightarrow 0 = 0.16T-20.8+0.89e^{0.058T}$$
$$\rightarrow 20.8 = 0.16T+0.89e^{0.058T}$$
$$\rightarrow Determine that for T=46.7 the condition d|a|/dT=0 is obtained.$$

B) Q. What is the acceleration in feet/seconds2 at this time?

## **Solution**

From answer to Problem2, the magnitude of the acceleration vector is

$${(36.6-0.284T)^2+(3.93e^{0.029T})^2}^{1/2}$$
... (1)

From answer to Problem3 A),

T=46.7 seconds...2

And "e" is Napier's constant, So "e" is  $2.718 \cdots 3$ 

From (1), (2) and (3), the magnitude of the acceleration vector is

$$= \{(36.6 - 0.284 \times 46.7)^2 + (3.93e^{0.029 \times 46.7})^2\}^{1/2}$$
  
= (545+228)<sup>1/2</sup>  
= 773<sup>1/2</sup>  
= 28

So answer is  $28 \text{ feet/sec}^2$ .

C) Q. If the acceleration of gravity at the earth's surface is 32 feet/sec<sup>2</sup>, how many 'Gs' did the astronauts pull at this time?

## Solution

'Gs' is unit of acceleration as 1 G=acceleration of gravity at the earth surface.

So 1G=32 feet/sec<sup>2</sup> in this problem.

In this time, acceleration of gravity is 28 feet/sec<sup>2</sup> from answer to B).

So  $G_s = 28/32 = 0.9$ .

So answer is 0.9 Gs.

## <Impressions>

Before going to Canada, I hadn't looked forward to going there because of a difficult task and since that was a study trip. But after finishing the study trip, I think this experience is essential and very useful for my future. So I want to treasure this experience.

-D. SATAKE

I think that the real pleasure of overseas travel is the people who live in there become not others for me. Before I going to Ottawa and Montreal, I had never think of that but I will do it from now on.

-Y. OHNISHI

I have experienced irreplaceable thing through this study trip. It is the difference of English between that I often hear and English of Canadian. From this failure, I want to take the advantage of this experience for the same kind of next things.

-T. NOZAKI

I could have experienced in Canada. It is very valuable experience to answer math question in English. While we translate English into Japanese, we must solve difficult question. It was very difficult.

—T. TAIRA

## < Problem#5 : Escape Velocity>

#### Miu KUMON, Eriko MAEDA, Ruri YOSHIDA

Questions:

a) Show that the escape velocity, v, of a small spherical object of mass 1 from the gravitational field of a body of a spherical mass M is given by

$$\mathbf{v} = \sqrt{\frac{2GM}{R}},$$

where R is the distance between the centers of the two bodies.

\*Reason

• Law Of Universal Gravitation

Supposed the object of mass m is in the gravitational field of a body of a spherical mass M, the strength of the power object pulled by body, F, is given by

$$F = \frac{MmG}{r^2}$$
 ... (See Figure 1)

•Newtonian Equation of motion

When the object of mass m is added the power of strength F and accelerating in acceleration a, F is given by

• Speed of object, v, is given by  $v = a \cdot t$  (Where a is object's acceleration and t is time) Then, differentialing both sides with respect to t,

$$a = \frac{dv}{dt} \quad \cdots \textcircled{3}$$

• Moving distance of Object, r, is given by  $r = v \cdot t$ Then, differentialing both sides with respect to t,

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\mathbf{t}} \quad \cdots \textcircled{4}$$

## **Solution**

a) Set the strength of power object of mass 1 escape from the gravitational field of a body of a spherical mass M as F.

From ①,

$$\mathbf{F} = -\frac{MmG}{r^2}$$

From (2) and (3),

$$\mathbf{F} = \mathbf{m} \cdot \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{t}}$$

Then,

$$-\frac{MmG}{r^2} = \mathbf{m} \cdot \frac{d\mathbf{v}}{dt}$$
$$\therefore \frac{GM}{r^2} = \frac{dv}{dt}$$

By the Chain Rule of differential calculus,

$$\frac{GM}{r^2} = \frac{dv}{dr} \cdot \frac{dr}{dt}$$

From (4),

$$\frac{MG}{r^2} = \nu \cdot \frac{d\nu}{dr}$$

Integrating both sides with respect to r, v is fixed when r=R. And when R= $\infty$ , v=0, we find

$$-\int_{R}^{\infty}\frac{GM}{r^{2}}dr = \int_{v}^{0} \left(v \cdot \frac{dv}{dr}\right) dr$$

Then,

$$-GM \int_{R}^{\infty} \frac{1}{r^{2}} dr = \int_{v}^{0} v \, dv$$

$$GM \left[\frac{1}{r}\right]_{R}^{\infty} = \frac{1}{2} [v^{2}]_{v}^{0}$$

$$GM \left(0 - \frac{1}{R}\right) = \frac{1}{2} (0 - v^{2})$$

$$\frac{GM}{R} = \frac{v^{2}}{2}$$

$$v^{2} = \frac{2GM}{R}$$

$$v = \sqrt{\frac{2GM}{R}} \quad (\because v > 0) \qquad \text{Q.E.D.}$$

b) Given the mass M, how small should the distance R be in order that the escape velocity v>c, where c is the speed of light?

Solution

Set v > c. From a),

 $v = \sqrt{\frac{2GM}{R}}$ 

So,

$$\sqrt{\frac{2GM}{R}} > c$$

$$\frac{2GM}{R} > c^{2} \quad \left( \because c > 0, \sqrt{\frac{2GM}{R}} > 0 \right)$$

$$\therefore R < \frac{2GM}{c^{2}}$$

c) Conclude from b) that if the body M were a star then such a body would have to appear "black" when viewed from the outside. This is an example of a "black hole".

#### Solution

Please look at Figure 2.

When we're opening our eyes, they are catching light which reflect from objects' surface and transmitting it to our brain. We call it "Seeing".

So, if we can see something not being black, it means that "photons could escape from its surface and reach our eyes" (like the left side of Figure 2).

In other word, when we can see an object, photons' speed (that always equal to the speed of light), c, is given by  $c \ge v$  (where v is the escape velocity from the object).

On the contrast, when v > c, photons can't escape from the object's surface (like the right side of Figure 2) and it seems be black.

(d) How small should the earth be in order for it to appear "black" to an outside observer?

### Solution

In this part, we're permitted to use the applet in the rejume (Figure 3. <u>http://hyperphysics.phy-astr.gsu.edu/hbase/vesc.html</u>). So we used it.

From part A,  $v = \sqrt{\frac{2GM}{R}}$ . So the smaller R is, the larger v is.

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From part C, v should be greater than c, the speed of light in order for it to appear "black" to an outside observer.

So we should shrink the earth, so we fixed  $M = 1M_{Earth}$  and try making r smaller until the escape velocity becomes bigger than the speed of light, about 299,800,000 m/s.

Then, we get the value. Maximum is about  $r = 1.3 \times 10^{-9} r_{Earth}$ .

The earth's radius is about 6,371 km, so

 $R = 6371[km] \times 1.3 \times 10^{-9} \approx 8.3[mm]$ 

So we would have to compress the earth to a radius of about 8mm if we wanted to make it looks black to outside observers.

<Mr. Mingarelli's Question>

How big is "8mm"?

----It's about 0.315inch. For example, there are movie films whose width is 8mm.

## <Impressions>

I'm not good at English and math. I could not understand a problem. But friends helped me. So I could cause a problem.

-M. KUMON

Although there were many troubles, I'm glad that we've managed to finish our presentation. However, if there was a next opportunity, it's better to determine the number of problems after determining students' groups. It was a bit difficult to handle two questions, and presentations took much more time than it was expected.

—E. MAEDA

I couldn't take part in the presentation.

Because I got sick and went to hospital in Ottawa city. I watched the video of the presentations in Japan.

Then I think that I wanted to join it. But I was glad that my group members and friends worried me. I felt I have good friends. I thought it was good my group members succeeded the presentation.

-R. YOSHIDA







A

Red





Figure 2



Figure 3

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# <Problem#6 : Fibonacci Sequence>

# Asuka TAKANAMI , Ayano TOKURA , Sora YANAGI , Keito HOTTA

(0),1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,.....

This sequence is the sum of one before number and two before number.

There are many examples in the Fibonacci sequence.

—This sequence exists many in nature world.

1. Pine cones and pineapple shadows.







- 2. Sunflower's face
- 3. Pascal's triangle



"Fibonacci" is name of the mathematician from Italy.

It is said that the ratio of the two numbers that are adjacent to each other in the Fibonacci sequence comes close to the "golden ratio" as much as possible.

"Golden ration" is the most beautiful ratio.



<Golden ratio>

Problem:

Show that if the following limit exists, then

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\phi\,,$$

where  $\varphi$ =1.618..... is the so called "Golden Number".

Solution

Define,

$$\frac{F_{n+1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$$

If,

$$\lim_{n\to\infty}\frac{F_{n+1}}{F_n}=\phi$$

Can be,

$$\lim_{n\to\infty}\frac{F_n}{F_{n-1}}=\phi$$

$$\frac{F_n}{F_{n-1}}$$

is one before term of

$$\frac{F_{n+1}}{F_n}$$

If,

$$\lim_{n \to \infty} \frac{F_n}{F_{n-1}} = \phi = L$$
$$L = \frac{F_n}{F_{n-1}} = \frac{F_{n+1}}{F_n} = 1 + \frac{F_{n-1}}{F_n}$$

$$L = 1 + \frac{1}{L} \rightarrow L^2 = L + 1$$
$$(\rightarrow \times L)$$

$$L^{2} - L - 1 = 0$$
$$L = \frac{1 \pm \sqrt{5}}{2}$$

Since L > 1,

We get  $L = \phi = 1.618 \cdots$ .

## <Impressions>

It was second time that I went to Canada.

First time, I don't have to speak English because my family can speak English.

But this time, I have to communicate by myself.

I worried about this problem but I understand that we can communicate without perfect English.

It was good trip for trying to speak English. Thank you very much.

—K. HOTTA

Irritation which doesn't speak the same language and the feeling of achievement when can communicate is very difficult and fun by Canada study travel.

A problem of mathematics also understood no beginnings. So while piling up discussion with a group member, it could be understood now gradually. Even if it could be understood in Japanese, the work which is corrected and told was difficult for English.

So after an announcement had ended, it was wrapped in the feeling of achievement. It was the travel which seems to would like to go to Canada.

-S. YANAGI

The meal in Canada is a great surprise for me.

Restaurants in Canada serves very large portions.

There are things to be not able to finish a meal.

Canadian will probably eat that amount of food every day, but I saw a plump person in Canada. It's mystery.

Buffet-style breakfast in the Hotel is very delicious.

This breakfast is the best I've ever eaten. I ate many salmon.

I can't speak English, but it worked out all right. Canadian are very kind.

Thank you very much.

—A. TAKANAMI

The trip to Canada this time was the first overseas in my life.

To tell the truth, I enjoyed myself but it was lonely.

It was my first time traveling by air on a long distance, so it was tired.

There were lots of surprises. Especially, I was most surprised about food.

It was delicious, but the amount was very large. Among them, pancakes and maple syrup were delicious. Also, the beaver tail was also delicious.

I was surprised at the university. It is wide and there is a food court in the university. The story of mathematics was also interesting.

It was a good experience, thank you very much.

The scenery was fun in a different atmosphere from Japan.

-A. TOKURA

# <Problem#7 : Barnard's Star>

## Shuhei KAWAMOTO, Naoyuki TAKEDA, Yuta MISAKI, Sho NAKASHIMA

Questions

The Close Encounter to the Sun of Barnard's Star

Problem1:

What is the distance to Barnard's Star at the present time?

## Solution

The parametric equations for the X and Y location of Barnard's Star can be approximated as follows:

$$X(T) = 2.0 + 0.00T Y(T) = 5.67 - 0.25T$$

Then, with the sun at the center of Cartesian coordinate grid, Barnard's Star can be represented as a point located at (2.0, 5.6) where the units are in light years. It is easy to see a triangle there, and we can use Pythagorean Theorem.

 $d = (2.0^{2} + 5.67^{2})^{1/2} = 6.0$  light years.

Problem2:

"What is the equation of the line Y=MX+B that is represented by the parametric functions?"

## **Solution**

According to Y'=M this linear function's slope is M.

*M*=*increment of y/increment of x* and we substitution

$$X(T) = 2.0+0.09T'$$
  
 $Y(X) = 5.07-0.25$ 

that is the parametric equations for the X and Y location of the Barnard's Star for M = *increment* of y/increment of x.

So

$$M = -0.25T/0.09T = -2.8.$$

And therefor

Y = -2.8X + B.

This linear function through the point (2.0, 5.6).



When we substitution this for that linear function, Y=11.2-2.8X.

Problem3:

At what time, T will Barnard's Star be closest to the sun along this trajectory? Solution

 $D(T)^{2} = (2.0+0.09T)^{2} + (5.67-0.25T)^{2}$ 

Take the derivative to get 2D,

$$dD/dt = 2(2.0+0.09T)(0.09)+2(5067-0.25T)$$

This simplifies to D,

dD/dt = -0.18 + 0.0081T - 1.42 + 0.0625T = -1.24 + 0.0706T

Set this equal to zero to get

0=-1.24+0.0706T.

Then T=17.6.

So in about 18,000 years from now, Barnard's Star will be closest to the sun. Its distance at this time will be

$$d^2 = (2.0+0.09*17.6)^2 + (5.67-0.25*17.6)^2 = 14.5$$

So d=3.8 light years.

## <Impressions>

We took a mathematics lesson at Carlton University in Ottawa. I was glad that I could communicate in English with foreign teacher. It was a rare experience for us. I would like to thank all of the people who have been involved with our trip.

-Y. MISAKI

What impressed me most about Canada was The Parliament Buildings.

I'm rather solemn and brilliant atmosphere unlike Japanese it.

In gift shop of The Parliament Buildings, I couldn't resist buying knit hat that used The Parliament Buildings.

Moreover, Both the supermarket I bought salad, the park squirrel lived, and Carleton Univercity I received the lecture is difficult for Japan. Therefore, I was surprised and moved.

However, I have a lot of places where I could not go. I want to visit again.

-N. TAKEDA

These problems were so difficult for me like the others math problems. However, I could try. That was because I didn't have to try to solve it by alone. So I could keep motivated.

Finally, I couldn't solve my problem, but thanks to Dr.Mingarelli, I had managed to understand. Through this travel, I could know that math is also interesting thing for me.

-S. KAWAMOTO

The trip for Ottawa was very nice memories for me. First I was so anxiety because I am not good at English. Bad I could talk naturally, and enjoyed. I was enjoyed the trip by shopping, eating, talking, sightseeing and such. My best impression place is Canada Parliament. I love the quiet and nice atmosphere place. The old wooden library of parliament was so beautiful. Other, I had have a lot of fun by the trip. The only problem is the shortest of trip days. So I want the next opportunity to visit Ottawa.

-S. NAKASHIMA

## <Problem#8 : Rotation Velocity of a Galaxy>

## Hironori NAKADA, Masaki TAKADA, Kouhei YAZAKI, Yuto YAMANAKA

\* Problem1: For small x (i.e., x < 1), what is the limiting form of V(x)?

## Solution

On the assumption that written on the paper.

Rotation speeds of stars can express this form of numerical expression.

$$V(x) = \frac{350x}{(1+x^2)^{\frac{3}{4}}}$$

We will have to think about the square of X in denominator, the square of X can dominates 0 so the denominator approaches 1. The function becomes 350X.

\* Problem2: For large x (i.e.x>1), what is the limiting form of V(x) ?

## **Solution**

According to the function, we will have to think about the square of X in denominator, the square of X can dominates over 2X, 2X will exceed 1. And it approach 1 to the limit. So, denominator would change this type.

$$V(x) = \frac{350x}{x^{\frac{3}{2}}}$$

We can divide X on denominator and numerator. So, we can reach this answer.

$$V(x) = \frac{350}{\sqrt{x}}$$

\* Problem3: The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)?

#### Solution

At a distance of 10,000 light years from the center, x=1.0. So, when the radius of M-101 is 90,000 light years, x=9.0. Therefore, the answer is,

$$V(9) = \frac{350 \times 9}{(1+9^2)_4^3} = 26km/s$$

\* Problem4: For what value of x is V(x) maximum?

#### Solution

At first, differentiate V(x).

$$V'(\mathbf{x}) = \frac{350 \times (1+x^2)^{\frac{3}{4}} - 350x \times \frac{3}{4} (1+x^2)^{-\frac{1}{4}} \times 2x}{\left\{ (1+x^2)^{\frac{3}{4}} \right\}^2}$$
$$= \frac{350 \left\{ (1+x^2)^{\frac{3}{4}} - \frac{3}{2} x^2 (1+x^2)^{-\frac{1}{4}} \right\}}{(1+x^2)^{\frac{3}{2}}}$$
$$= \frac{350 \left( 1+x^2 - \frac{3}{2} x^2 \right)}{(1+x^2)^{\frac{3}{2}} \times (1+x^2)^{\frac{1}{4}}}$$
$$= \frac{350 \left( 1 - \frac{1}{2} x^2 \right)}{(1+x^2)^{\frac{7}{4}}}$$

So

$$V'(\mathbf{x}) = 0 \Leftrightarrow 1 - \frac{1}{2}x^2 = 0$$
$$1 - \frac{1}{2}x^2 = 0$$
$$\mathbf{x} = \pm\sqrt{2}$$

So the increase and decrease list of V'(x) is as follows.

х	0		$\sqrt{2}$	
V´(x)	/	+	0	Ι
V(x)	/	Î	maximum	↓

So V'(x) takes the maximum at value of  $\sqrt{2}$ .

\* Problem5: For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals  $9.5 \times 10^{12}$  km, and there are  $3.1 \times 10^{7}$  seconds in a year?

## **Solution**

For x=1 the physical distance is 10,000 light years. And the distance of 1 light year is  $9.5 \times 10^{12}$ . So, when x=1 and  $\pi$ =3.14. Length of the galactic lap is

$$2\pi R = 2 \times 3.14 \times (9.5 \times 10^{12} \times 10,000)$$
  
= 6.28 \times 9.5 \times 10^{16}  
= 5.9 \times 10^{17} (km)

Again the galactic orbital velocity at x=1 is 208km/s from a problem sentence. So, when x=1, the rotation velocity of galaxy is

$$\frac{5.9 \times 10^{17}}{208} = 2.8 \times 10^{15} (s)$$

Because it is  $3.1 \times 10^7$  second for one year.

$$\frac{2.9 \times 10^{15}}{3.1 \times 10^{17}} = \frac{29 \times 10^8}{31} = 9.3 \times 10^7 (y)$$

## <Impressions>

I was very excited at Canada trip. I am writing this passage when I passed three month from Canada trip but I can remember experience from there. I think biggest influence of this trip is considering the math in English. I am not good at considering Math because math needs ability about consider in developing. But this trip help me to require the ability not only changing difficult English into easy English but also consider math idea in English.

By the way, I thought one problem of this trip is not enough time to rest. In this trip some people made sick. So I think we should make enough rest time such as when we change the plane. But to see the total this trip will be one of the best experience in my life.

## -Y. YAMANAKA

I had good experience during my stay in Canada. In a complicated situation, I went to a police office in there. There were many people whose races were various. This is because, Canada is a multiracial country which welcome emigrants. I could make certain those circumstance in there, and I was moved because in USA, president Trump had decided to reject emigrants from particular countries. I had been against this decision, so this discovering in there conforted me greatly. I would like to tell my experience to many other people.

-K. YAZAKI

Thank you for inviting us. AII of views which I saw in Canada was very flesh. And I felt that I challenged our presentation was so difficult, but I learned a lot this time. So, I want to make use of this experience for my life in the future

## —H.NAKADA

I was enjoyed during I was in Ottawa. In that the most impressive building is the Diet building. It was very big and was the building such as the castle. In addition, I had very good experience that study math in English. I like math, so study math in English was very intesesting. I want to go to Ottawa again.

-M. TAKADA

# <Problem#9 : Redo Things >

## Daiki TERADA, Syunsuke MOCHIZUKI, Yudai UEMURA

Question : why are hot things red?

## **Solution**

First of all, this problem is related to color temperature. Color temperature is the numerical value of light color. The unit uses Kelvin. As shown in the figure, the color becomes red when the Kelvin is high and it is law it turns white.

To calculate the color temperature, we use Planck's law and Wien's displacement law.

Today we are going to prove Wien's displacement.

At first make a graph of temperature and peak wavelength. Figure1 is the sheet we'll use this time. We are going to find the graph match to this sheet. First we regarded the temperature as T and the peak wavelength as  $\lambda$ . Next, find the constant. So we fixed T for 300K.

First we'll find a which is the constant of liner function  $\lambda = aT$ . Figure2 is the graph of this liner function. If we find B which is the constant of invert propotion  $\lambda = bT^{-1}$ , result becomes similar to figure4. In the same way we find  $\lambda = cT^2$ ,  $\lambda = dT^{-2}$ ,  $\lambda = eT^3$  and  $\lambda = fe^{(gT)}$ , results are becomes as figure3, 4, 5 and 6.

There are only  $\lambda = bT^{-1}$  that satisfying the conditions. So the Wein's displacement's graph's form becomes invert proportion. At this time, the constant b is 2898.

Next, we try to solve calculus problem Find Equation and Find the solution to Equation. First we divide this fraction( $A/\lambda^{5e}$  (14329/( $\lambda$  T))-1 ) into two. By putting A  $\lambda^{-5}$ =U, and  $e^{(14329/(\lambda T))} - 1 = V$ .

Next, we try to differentiate by U and V. As a result, U changes to  $-5A\lambda^{-6}$ , and V changes to  $-14329 / (\lambda^2 T) e^{14329/(\lambda t)}$ . By means of the quotient rule, we calculate  $d\lambda/du(U/V)$ . As a result, it change to  $1/V \times d\lambda/dU - dU/V^2 \times d\lambda/dV$ . The derivative equal to Zero, so  $d\lambda/dU - U/V \times d\lambda/dV = 0$ .

We substitute this formula for U and V. Thereby this formula is changed

 $51T(e^{14329/(\lambda T)} - 1) - 14329e^{14329/\lambda T} = 0$ 

Then by putting  $14329/\lambda T = X$ ,  $5(e^{x} - 1) - xe^{x} = 0$ .

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Next, solve for the value of X. This formula's  $(5(e^x - 1) - xe^x)$  answer is 0. Then trial and error, we find out that the value of the X is 4.965. Therefor,  $4.965 = \lambda T/14394$ . So,  $\lambda = 2899/T$ . From the above, Wien's displacement law is true.

For example, the temperature of a magma is about 1500K. With a law, a magma is red. In this way, the color of things is decided by Wien's displacement law.











Figure 2



Figure 4

Figure 6



Figure 5

Peak
Wavelength
(microns)
0.2898
0.322
0.362
0.414
0.483
0.579
0.724
0.966
1.449
2.828
5.796
9.660

Table of a base

## <Impressions>

This trip was my first experience, so I got nervous very much. I didn't go well, but I could enjoyed. I appreciate our teachers and Mr. Mingarelli. I will not forget it.

-D.TERADA

While I went to Canada I was sleeping with poor physical condition, but still I think I did a lot of good experience. Especially the class at the Carlton University was explained carefully so that I can understand even I who is not good at mathematics.

I would like to encourage studying by making use of this experience.

-Y.UEMURA

This trip to Canada was the my second travel to abroad. At the first travel, I was 12 years. So, I can't remember about the diffent point between foreign and Japan. But this time, I've already been 17, so I identified Canada has a lot of diffrent point than Japan. For example road signs, they were written by English and French. I'm not used to see French. This thing was the best point which I was reaized that I am in Canada. Totally, this trip becomes my precious experience.

-S.MOCHIZUKI

# <Problem#10 : Prime Numbers >

# Kazuya AWAZAWA, Masashi TOKOYODA

## Question

Define an arithmetic progression. State the Green-Tao Theorem for primes in arithmetic progression. For n = 3; 4; 5; 6; 7 find an arithmetic progression containing n primes. What is the largest value of n known so far for which there are n primes in an arithmetic progression?

## Solution

- 1) Define an arithmetic progression as a sequence of numbers such that the difference between the consecutive terms is constant.
- 2) We found arithmetic progressions of n primes/ n=3, 4, 5, 6, 7...

$$n=3 \quad ``7, 13, 19"$$

$$n=4 \quad ``41, 47, 53, 59"$$

$$n=5 \quad ``5, 11, 17, 23, 29"$$

$$n=6 \quad ``7, 37, 67, 97, 127, 157"$$

$$n=7 \quad ``7, 157, 307, 457, 607, 757, 907"$$

Also, we found that we have to use a number which has a product of primes included from 1 to n when we make a progression whose term number is n.

- 3) The largest value of n known so far for which there are n primes in an arithmetic progression is 26. And its first term is 161004359399459161, and its difference is 47715109 223092870(This is an multiply of 30).
- 4) The Green-Tao Theorem is simply put, "We can make any length of Arithmetic progression". However, the longest length of Arithmetic progression which it has been found is 26. It is wonder!!

## <Impressions>

It took a lot of time for preparing this presentation. Kazuya and I discussed the problem and researched things related to it so many times. Unfortunately, I was not able to participate in the presentation of the performance due to my illness. However, Kazuya managed to do it. I would like to take this opportunity to express my appreciation to him. We had so much difficulty in tackling the problem, but this experience is very special and precious to us. Also, I would like to my sincerest thanks to Dr.Mingarelli, Mr.Akiyama, Ms.Ono, Mr.Shirakawa, my friends and my precious family.

#### -M. TOKOYODA

As Masashi says, this program was very difficult. We took a long time to prepare for presentation. However, it was not complete one. And because of unlucky accident, I came to do this presentation by myself. I was very impatient. And we couldn't make complete script. After all, I managed to do half of this presentation in an ad lib. It was very hard work, but I gained very good experience. I say it over and over, this problem is hard for high school students. So we tried to pick up "What we can understand" and tried to think it. "I couldn't understand" in other part in the presentation. I think this was very important. It certainly was hard, but it was very enjoyable to convert my idea into English. And I was glad when I could talk with Canadian in my English. Finally I would appriciate to all people who supported this research trip, Mr.Akiyama, Dr.Mingarelli, Mrs.Ono, Mr.Shirakawa, and all of my friends who participated in this trip. Thank you!

-K. AWAZAWA

# PICTURES

Parliament building, capital town, Sugar bush , maple syrup , taffy , lecture, presentation, Around OTTAWA short trip (Montreal)









Group photograph at Haneda Airport.



A flight to Ottawa,time difference is thirteen hours.

We will be looked after by Mr.Shirakawa.



Ottawa airport is spacious!!



Buffer breakfast at a hotel is splendid.



Yippee! snow!!

Peace sign!!



Everyone is excited about Canadian snow.



High school students are romping around.



A magnificent building ...

Discover a Canadian snowman!!



The charming place is exist.



A library that tickles our curiosity.



From a observatory of the Houses of Parliament.

The guide is a beautiful woman.



Fatigue of long moving hours began to show...



Beacon, sausage and bean!!





Which of the menu do I like?



They harden maple syrup with snow.



It's extreme luxury to use natural snow.





It is finished.



It's a panel that someone fit their face.



The back of the Houses of Parliament!!



It's very spacious!!

They looked sleepy.



It's a magnificent where I feel comfortable.



The good streets and houses





To Carleton University by a city bus





Professor Mingarelli's lecture.



A student of science did their best.



Making summary was pretty tough.

We worked on math assignment from winter vacation.



A student of humanities of social sciences also did their best.



Everyone is real sword.



Speaking English is tough...



I was nervous that I speak in front of a large audience.

# 

Teacher teached slowly.



The announcement a little more.





They are received teacher's comment.

We are also asked by teacher.



It's time to show the results.



Every group's level is high.



NE



Pinch-hitter appears because his parner is sick.



I'm hungry.



It's dinner at the last!!





I'm looking forward.



Energetic people.



There were spacious and stylish.



They become more and more fatigued.

This picture show a dynamic and uplifting feeling.



This beaver tail is very big!!



Wow,that looks delicious! fatty!!





We order the menu ourself.



This restaurant is expensive.



We have food that we order each.



Building are standing in a row. (Montreal)



I'll be back, Thank you!! (OTTAWA Airport)

Beautiful weather!! (Montreal)



# <Impression of Study Trip to Canada>

Eriko OHNO(mathematics teacher)

## 1.Foreword

Last year, several second year students gave a presentation in the auditorium during home room period about a study trip to Canada. They were able to get the trip approved because there were 30 students who were interested following the presentation. This study trip required the students to do two things. One was that the students were supposed to attend math lectures at Carleton University. The other was that they had to interact with the math professor during the lectures. These requirements were hard, but there were 30 students who wanted to participate in the trip, which showed that there were more students showing high motivation than we had expected.

As a prior learning exercise, Dr. Angello Mingarelli, a professor of Carleton University, provided 10 questions relating to mathematical and scientific fields. It took as long a full half year for the students to solve them as a group, which was partly because the students had to give a presentation in English for Dr. Mingarelli at Carleton University. In order to do that, the students had to do many things like reading in English, understanding mathematics, solving the mathematical problems, and thinking about what they were going to do during the presentation. After the students returned to Japan, the students were required to make a final report for their journal. My focus will be to report on what the students did and learned before and after the trip to Carleton University.

#### 2. Before the trip

In late September, the members of the study were chosen and we asked Dr. Mingarelli to provide them with ten tasks that they were required to complete. We informed Dr. Mingarelli of what our students had learned, that is, trigonometric functions, vectors, sequences, and calculus. However, what he gave them turned out to be beyond the scope of high school mathematics. It also included a wide range of science and astronomy, topics which were considerably challenging for the students.

We divided the members into 10 groups, each group decided on its task and began to read the task in English. However, it was difficult for the students to comprehend what they had not learned in their math classes. That is to say, even if they translated the task into English, they had trouble understanding it. Consequently, we decided to hold a study session after school on Thursdays. Specifically, the students who were good at mathematics gave calculus lessons, learned the mathematical terms and expressions, and then spent time acquiring the basic math necessary for each group to solve their problems.

It wasn't until December that the students started to understand their tasks through the study sessions completed after school. Additionally, during the winter vacation, through the guidance of Mr. Akiyama and the intergroup cooperation, they managed to solve half of the problems by the third term.

During the 3<sup>rd</sup> term, before the study trip, the students began to work on the presentation manuscript. Regardless of the language, we can share the content by presenting the formulas. Therefore, our challenge was how to combine the parts which would be communicated by words, the parts which would be presented by formulas, and the parts which would be explained by the charts. Each group had meetings on that topic and completed it. In March we had a rehearsal and we left for Carleton University on March 26<sup>th</sup>, 2017.

## **3.Study Trip Itinerary**

March 26<sup>th</sup> (Sunday)

17:40 Departed Haneda Airport for Ottawa Airport via Toronto International Airport

21:10 Arrived at Ottawa Airport and proceeded to the hotel

The flight was 12 hours, which was longer than the students were prepared for. There were more problems than we had expected with the flight. Some students became air-sick, had stomach problems from the airline meals. Also, one student left her mobile phone on the airplane, which was luckily found by a cabin attendant. Another student lost her passport and had to have it reissued. The time lag between Canada and Japan was 13 hours at this time, so as soon as we reached the hotel at 10 p.m., we were supposed to go to bed, but many students said that they had trouble going to sleep. The next day many students said that they didn't feel well due to lack of a sleep.

March 27th (Monday) Spent all day sightseeing in Ottawa

We charted a bus until 4 p.m., and visited Capitol Hill and a Canadian History Museum in Ottawa. We also observed the process of making maple syrup at Sugar Bush and had pancakes with syrup. The syrup was a little bit too much for some of the students who weren't feeling well. During the guided tour, we heard the explanation that maple syrup is widely used

because it has a low calorie. It is not only put on food as a sauce but also used like *mirin*, which is a sweet cooking rice wine. The students were surprised to see the container because it was one liter, which is rare in Japan.

After going back to the hotel, the students were given about an hour of free time. Some students who did not like the local food said that they wanted to have salad or vegetables, so some of them went to a supermarket near the hotel. Other students also enjoyed shopping near the hotel. Ottawa is both safe and bright at night, so I thought that it was a good choice for a school trip for high school students who enjoy going out on their own, which meant that we could increase the amount of freedom for them.

March 28th (Tuesday) Morning - trial class at Carleton University

Afternoon - students' free time in Ottawa

The students then made their presentation at Carleton University, which was the main purpose of the study trip. Many students stayed up late at night practicing, so they didn't get enough sleep. However, they were very energetic, although they felt nervous.

At first, Professor Mingarelli gave a lecture to the students, and then each group made their presentations. Even though they did not understand the details of the lecture, it seemed that the students grasped the synopsis depending upon the formulas or charts on the blackboard. I think that mathematics is suitable for the students to use because of the English that they are learning.

The students' presentations were much better than I expected, as they had spent half a year preparing for it. As a matter of fact, Dr. Mingarelli appreciated it highly. I would appreciate it if you refer to the summary of the students' presentations included at the end of this document. After each presentation, Dr.Mingarelli on the information that was presented. The students did not know what questions were given beforehand, so they had to answer them on the spot, but most of the groups managed to do that while they were consulting one another because they had worked on the task seriously. I was impressed that they were able to interact with the professor.

In the afternoon students were supposed to have a group activity during their free time, but some students took a nap because they were tired from their presentations in the morning. The students who went out seemed to spend time shopping at the shopping mall or the supermarket.

At dinner, we went to a nearby restaurant, and each student ordered a meal and paid for it by himself or herself. In fact, they were supposed to do all the things like asking where the washroom is, getting water, ordering the dessert, paying the bill, without depending upon the teachers as much as possible. As a result, they were able to do these things by themselves. In comparison to the first day of the trip, the students had become more active, and they seemed to have changed in the way they were enjoying the trip.

March 29<sup>th</sup> (Wednesday) Spent all the day sightseeing in Montreal

We went to Montreal on the bus, saw the Norte Dame Church, the old city, the Mon Royale Hill, and Saint Joseph Cathedral. The atmosphere in Montreal was very much a French culture which was very good for the students to experience.

March 30<sup>th</sup> (Thursday) Returned to Japan via Toronto from Ottawa

March 31st (Friday) Arrived at Haneda Airport, Japan

From the flight experience of going to Canada, some students tried not to eat the airline meals by purchasing a sushi roll or choosing not to have the inflight cup noodle offered at midnight instead. The students did not feel sick because they took care of themselves on the plane. Due to jet lag, some of the students could not sleep well arrived in Japan, but I was relieved that I could take my students back to their families.

However, at the passport control, one student who lost his passport was not allowed to go through, and had to wait outside the gate. In the end, his passport was found and he came out of immigration. If the study trips abroad are planned, it is essential to teach the students thoroughly the importance of keeping their valuable items. In the future I am afraid that there will be a student who cannot go back to Japan because they might lose their passport or other important things.

## 4.Conclusion

After this trip I realized that it is necessary to provide the opportunity to foster zest for living for our students and put what they have learned into practice. I was surprised that there were so many students who were not able to get the things around them tidy and take care of themselves, which I don't think has anything to do with their academic ability. They will lose their belongings while traveling if they are not careful. I also felt uneasy when I saw the students looking for their passports by putting down and opening their baggage every time. If we go to an unsafe foreign country, we teachers will be very nervous.

I worry that they don't have the ability to be proactive. On the first day the students who were hesitant to speak English realized that they had to use it on their own because their

requests were not passed if they did not speak. I saw this when they began to speak to the clerks and flight attendants. They started learning English in junior high school, but it might be difficult for them to get enough opportunities to use English if they only use it at school.

The purpose of this study trip was to learn mathematics through English, but it seemed that the students also learned the importance of being active and realized what was most necessary for each of them. I hope that they are a little more mentally tough and more conscious about learning. For example, I am very happy if they learn that just getting a high score is not real learning. If they felt the joy of active learning a little, they might not only learn the things which are taught to them but also learn things actively on their own, and change their ways of thinking about their future. I hope that this study trip will be stimulate their awareness.

> 平成 29 年 9 月 19 日 (株) JTB コーポレートセールス 法人営業東京多摩支店 営業担当:白川 佳弘

~ 中央大学附属高等学校カナダ研究旅行に同行して~

平成29年3月26日(日)~31日(金)の6日間、団長秋山先生・大野先生他 中央大 学附属高等学校の30名の生徒の皆さんとカナダの首都であるオタワへの研究旅行に添乗さ せて頂きました。

当初予算面が非常に厳しく社内でもツアー催行に反対意見もありましたが、秋山先生の 「オタワで生徒達に本物の数学とは何かを見せてあげたい。カナダの大自然などを生徒にも 体験させたい。」という熱意に打たれ、このツアーを催行するべく必死で社内を説得しました。 (結果的に上司の交渉のおかげでツアー催行にこぎつけました。)

年明けから生徒の皆さんは秋山先生の課される設問に取り組むとともに、弊社より依頼し た渡航手続き書類の作成やパスポートの取得を行って頂きました。全員が期日をきちんと守 られて、出発前最終説明会もスムーズに進みました。 (ツアー中生徒の皆さんには、集合時間の厳守や持ち物チェックなど口うるさく言ってし まいました。申し訳ありませんでした。でもみんな集合時間に遅れることもなかったですし、 大きなトラブルもなかったように思います。)

出発当日、15時30分 羽田空港国際線ターミナル集合。全員予定よりも早めに集合完了。 春休みということで、結構一般のお客様がいらっしゃいました。搭乗までの説明をし、エアー カナダのカウンターでチェックイン+荷物預け等の手続き。みんなスムーズに進んでセキュ リティーチェック・出国手続き。順調そのものでした。

17時40分発エアーカナダ006便で一路トロントへ。機内では皆さんそれぞれでくつろが れていましたね。16時45分トロント空港到着。カナダの入国手続きを済ませて国内線の乗 り継ぎへ。夕食用のお弁当と水も現地ガイドさんより受け取れて、無事国際線のターミナル へ。みんなもう手慣れたものでした。

19時10分発オタワ行きのエアーカナダ464便無事離陸。20時10分頃オタワ空港到着。 この日は宿泊するホテルロードエルギンへ直行しました。

翌日(現地時間3月27日)はオタワ市内観光。国会議事堂・カナダ歴史博物館・シュガー ブッシュ(昼食で名物のパンケーキを食べる)・ガティノパークを回りました。カナダの首 都だけあって国会議事堂など歴史的な史跡もたくさんあり、皆さんしっかり勉強されていた ようです。昼食もボリューム満点だったとお聞きしています。私は、パスポートを紛失され た生徒さんと現地の警察、日本大使館を周り、日本のご両親に連絡し書類を手配して頂いた りと別行動を取っていたため夕食から合流しました。ちょうど現地が月曜日、日本が火曜日 の平日だったため手続きを1日で終えることができ、30名全員で帰国できたことは幸運だっ たと思っています。海外旅行ではパスポートは一番大切な持ち物となります。海外に行かれ る際には、パスポートは肌身離さず携帯するようにして下さい。

夕食はホテル周辺で市場の近くにあるレストランにて、魚料理を食べました。お昼のパン ケーキが残っていたため、完食できた生徒さんはほとんどいらっしゃらなかったです。

3月28日は秋山先生の案内のもと、オタワにある有名大学カールトン大学に行きました。 こちらで先生の師匠であるアンジェロ・B・ミンガレロ教授の体験授業に参加しました。 この日のために、生徒の皆さんは事前学習課題をこなされてきました。体験授業中も積極的 なやりとりがなされ、有意義な経験ができたと秋山先生よりお聞きしました。

この日私は、体調不良の2名の生徒様とともに病院へ行っていました。2名が診察を終え て薬をもらうまでおよそ2時間半かかりました。日本の病院がいかに患者さんのことを考え て診察にあたっているかよく分かりました。 タ食は近くのレストランで洋食を各自オーダーして食べました。パスタ系を食べる人、果 敢に英語を駆使して料理を注文する人などそれぞれでした。みんな徐々に英語が使えるよう になっていましたね。

3月29日、オプショナルツアー「モントリオール1日観光」に皆さん参加されました。 ケベック州最大の都市で、パリに次ぐ世界で2番目に大きなフランス語圏の都市である モントリオール。丘から見た景色と気候から、北海道の札幌の町並に似ていると私自身は感 じました。

ノートルダム教会やモンロワイヤルの丘・旧市街観光などを行いました。この日の昼食は 秋山先生よりご希望を頂いたフランス料理フルコース(鴨肉中心)をいただきました。先生 から「本物のフランス料理を出してくれないとダメだからね」とのお言葉を頂き、レストラ ンの選定にはJTBモントリオール支店の担当マネージャーと何度も折衝をしました。出て きた料理はなかなかのもので、先生からも何とか合格点を頂けたように私は感じています。 タ方ホテルに到着し、ホテル内のレストランで夕食をとり翌日の帰国の準備を生徒の皆さん は各部屋で行われていました。

3月30日、オタワ出発の日。ホテルで朝食を済ませて一路空港へ。到着後エアーカナダ のカウンターでチェックイン。皆さんもう海外旅行は手慣れたもので、スムーズにチェック イン・セキュリティー・出国審査を終えられていました。10時オタワ空港発のエアーカナ ダ447便にてトロントへ。トロントで乗り継ぎを行い、13時45分発エアーカナダ005便に て一路羽田空港へ。3月31日15時30分頃に予定通り羽田空港に到着。空港到着後、生徒 の皆さんより嬉しいお土産を頂きました。ありがとうございました。

4泊6日の行程を振り返ってみて、この旅行中生徒の皆さんのご協力と秋山先生・大野先 生の的確な指導のおかげで無事帰国できたと感じています。6日間という短い期間ではあり ましたが、旅行中語学力や国際感覚が養われていく生徒の皆さんの姿を見ることができ、私 自身この研究旅行に携わることができたことを誇りに思いました。この旅行を通じて是非参 加した皆さんの中から、世界にチャレンジしていくグローバルな人材が一人でも多く輩出さ れることを祈願してなりません。

最後になりましたが、生徒の皆さんの今後のご活躍を陰ながら応援させて頂きます。

以 上

平成 28 年 12 月 17 日

## 中央大学附属高等学校平成 28 年度研究旅行日程表

カナダ:オタワ・カールトン大学体験授業 エアカナダ利用

平成29年3月26日(日)~31日(木) 参加人数:生徒30名様+ご引率2名様+添乗員1名

	月日	地名	現地時間	交通機関	行程	食事
1	3/26	羽田空港集合				タ:機内
	(日)	羽田空港発	17:40	AC006 便	カナダにて入国審査が必要になります。	
		トロント空港着	16:45		トロント乗換時、お弁当+飲み物を配布予定	昼:機内
		トロント空港発	19:10	AC464 便	14	
		初空着	20:21		弊社係員がお出迎えし、専用バスで市内のホテルへ	夕:0
		オタワ	21:30		(宿泊ホテル:ロールエルギン)	
2	3/27			貸切バス		餌・○
_	(月)			利田		171 · C
	()/	オタワ		(8時間)	午後:Sugar Bush+Civilization museum	■ 尽・×
					目学 (昼食け冬白 Sugar Buch でお取り頂き	<b>E</b> . A
				9:00.0	北子 (宣反は古日 bugar busit Cloury ) (Course ) すす 支払けを白)	Ø · O
				17:00	(定泊ホテル・ロールエルギン)	2.0
2	2/28			政治バフ		
5	(Jk)	+ 4 17		(タウェゼ)	朝良夜、路線ハスビルールトン人子へ。	朝:〇
		4 9 9		(各自文位)	午前中:仲糠党美	
					(字校様手配。特別な費用は見込んでおりません)	昼:×
					大学のカフェテリアで各自昼食後、	<b>H</b> V
					市内中心部で自由行動。	タ:×
					(宿泊ホテル:ロールエルギン)	
4	3/29			貸切バス	朝食後、モントリオールへ。	朝:〇
	(水)	オタワ			★モントリオールオプショナルツアー(昼食付)	
					全員参加にてモントリオール見学	昼:〇
					昼食は洋食料理を召し上がって頂きます。	
					日本語ガイドサービスは、モントリオールのみ	夕:〇
					同行。 (宿泊ホテル:ロールエルギン)	
5	3/30	オタワ発	07:30		ホテルで朝食後、専用バスで空港へ。	朝:〇
	(木)	オタワ空 港 着	08:00		乗り継ぎで帰国の途へ	
		オタワ空港発	10:00	AC447 便		昼:×
		トロント空港着	11:20		(機中泊)	
		トロント空港発	13:45	AC005 便		夕 : 機内
6	3/31	羽田空港着	15:35		羽田空港着	
	(金)					夕:機内

	3/26 (日本)	3/26 (日本)	3/27	3/28	3/29	3/30
朝食						
	×	×	ビュッフェ	ビュッフェ	ビュッフェ	ビュッフェ
昼食						
	×	機内食	×	×	<b>Oレストラン</b>	機内食
夕食				-		
	機内食	お弁当+飲物	<b>Oレストラン</b>	×	<b>Oレストラン</b>	機内食

オタワ市内見学候補地=国会議事堂(入場観光)/国立美術館(入場観光)/カナダ造幣局(見学)

ノートルダム大聖堂(入場観光)/コンフェデレーション広場(車窓・徒歩通過見学)/

リドー運河(車窓・徒歩通過見学)・ネピアンポイント(車窓・徒歩通過見学)

カナダ文明博物館(入場観光) ★時間帯を考慮しながら見学ルートを決めていきます。

モントリオール見学候補地=パルク・デュ・モン・ロワイヤル/ドーチャスター広場/ノートルダム教会/サンジョセフ礼拝堂

(下車観光)/世界の女王マリア大聖堂(入場見学)/サンポール通り・サンローラン通り(自由行動)

★時間帯を考慮しながら見学ルートを決めていきます。

# < Conclusion >

This was the third Canada study trip. Every time, the number of participating students has increased and this time it was up to 30 students. Professor Mingarelli, my supervisor, in the Department of Mathematics at Carleton University who accepted and cooperated with the program in many ways. I would like to express my heartfelt gratitude to Professor Mingarelli.

It is in the student's own mind whether 12 months were long or short after the preparation period. I had pleasant thoughts thinking about mathematics in English when I was studying abroad forty years ago. Furthermore, it is obvious that speaking English is not equivalent to globalizing Japan, but like money, it is better to have something rather than not, meaning it is better be able to speak English rather than not. I wanted my students to present their solutions without script.

English is a very good companion to mathematics using logic as the main weapon, and it helps to understand mathematics. If something is announced in English, 4 billion people around the world can understand it. However, if announced in Japanese, it would be at most 100 million people. Fortunately, this was a chance to think about what true globalization is.

Finally, as long as there are students participating in the Canada study trips, I strongly recommend continuing the program.